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Origins of the Quark Model*

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ABSTRACT

An intellectual history of the quark model prior to February 1964 is presented. Aspects of this history are best summarized by a parable:

Man asked God for a riddle, and God obliged:

"What is green, hangs from a tree, and sings?"

This, of course, was a very difficult question.

So man asked God for the answer, and God replied:

"A herring!"

"A herring? But why is it green?"

"Because I painted it green."

"But why does it hang from a tree?"

"Because I put it there."

"And why does it sing?"

"If it didn't sing you would have guessed
it was a herring."

When Nathan Isgur first asked me to talk about the early history of the quark model, I was reluctant. Seventeen years had passed. Even important events were hard to remember. In addition, the negative reaction of the theoretical physics community to this model when I first proposed it had left a lingering unpleasant aftertaste. However, after looking over my notes and rereading early papers, I was able to reconstruct the spirit of that time, the ideas and influences acting on me. This history is worth recording.

The quark model had two independent births. I will describe the one I witnessed. Perhaps Murray Gell-Mann will one day illuminate the other.

The history of the quark model has no clear beginning. One natural starting point is the remarkable 1949 paper of Fermi and Yang who discussed the possibility that the recently discovered π meson was not an elementary particle, but rather a nucleon-antinucleon composite:

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 76, NO. 12

DECEMBER 15, 1949

Are Mesons Elementary Particles?

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(Received August 24, 1949)

The hypothesis that π -mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

I. INTRODUCTION

IN recent years several new particles have been discovered which are currently assumed to be "elementary," that is, essentially, structureless. The probability that all such particles should be really elementary becomes less and less as their number increases.

It is by no means certain that nucleons, mesons, electrons, neutrinos are all elementary particles and it could be that at least some of the failures of the present theories may be due to disregarding the possibility that some of them may have a complex structure. Unfortunately, we have no clue to decide whether this is true, much less to find out what particles are simple and what particles are complex. In what follows we will try to work out in some detail a special example more as an illustration of a possible program of the theory of particles, than in the hope that what we suggest may actually correspond to reality.

We propose to discuss the hypothesis that the π -meson may not be elementary, but may be a composite particle formed by the association of a nucleon and an anti-nucleon. The first assumption will be, therefore, that both an anti-proton and an anti-neutron exist, having the same relationship to the proton and the neutron, as the electron to the positron. Although this is an assumption that goes beyond what is known experimentally, we do not view it as a very revolutionary one. We must assume, further, that between a nucleon and an anti-nucleon strong attractive forces exist, capable of binding the two particles together.

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We assume that the π -meson is a pair of nucleon and anti-nucleon bound in this way. Since the mass of the π -meson is much smaller than twice the mass of a nucleon, it is necessary to assume that the binding energy is so great that its mass equivalent is equal to the difference between twice the mass of the nucleon and the mass of the meson.

According to this view the positive meson would be the association of a proton and an anti-neutron and the negative meson would be the association of an anti-proton and a neutron. As a model of a neutral meson one could take either a pair of a neutron and an anti-neutron, or of a proton and an anti-proton.

It would be difficult to set up a not too complicated scheme of forces between a nucleon and an anti-nucleon, without about equally strong forces between two ordinary nucleons. These last forces, however, would be quite different from the ordinary nuclear forces, because they would have much greater energy and much shorter range. The reason why no experimental indication of them has been observed for ordinary nucleons may be explained by the assumption that the forces could be attractive between a nucleon and an anti-nucleon and repulsive between two ordinary nucleons. If this is the case, no bound system of two ordinary nucleons would result out of this particular type of interaction. Because of the short range very little would be noticed of such forces even in scattering phenomena.

Ordinary nuclear forces from the point of view of this theory will be discussed below.

Unfortunately we have not succeeded in working out a satisfactory relativistically invariant theory of nucleons among which such attractive forces act. For this reason all the conclusion that will be presented will be

extremely tentative.

When their work was first described to me I thought it was obviously wrong. By then the antiproton had been discovered. Its interaction with a proton was clearly not strong enough to form a nearly massless pion. I didn't read their paper.

The Sakata model, which extended these ideas in 1955 after the discovery of strangeness, did not seem much better:

Progress of Theoretical Physics, Vol. 16, No. 6, December 1956

On a Composite Model for the New Particles*

Shoichi Sakata

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University, Nagoya*

September 3, 1956

Recently, Nishijima-Gell-Mann's rule¹⁾ for the systematization of new particles has achieved a great success to account for various facts obtained from the experiments with cosmic rays and with high energy accelerators. Nevertheless, it would be desirable from the theoretical standpoint to find out a more profound meaning hidden behind this rule. The purpose of this work is concerned with this point.

It seems to me that the present state of the theory of new particles is very similar to that of the atomic nuclei 25 years ago. At that time, we had known a beautiful relation between the spin and the mass number of the atomic nuclei. Namely, the spin of the nucleus is always integer if the mass number is even, whereas the former is always half integer if the latter is odd. But unfortunately we could not understand the profound meaning for this even-odd rule. This fact together with other mysterious properties of the atomic nuclei, for instance the beta disintegration in which the conservation of energy seemed to be invalid, led us to a very pessimistic view-point that the quantum theory would not be applicable in the domain of the atomic nucleus. However the situation was entirely changed after the discovery of the neutron. Iwanenko and Heisenberg²⁾ proposed immediately a new model for the atomic nuclei in which neutrons and protons are considered to be

their constituents. By assuming that the neutron has the spin of one half, they explained the even-odd rule for the spins of atomic nuclei as the result of the addition law for the angular momenta of the constituents. Moreover, they could reduce all the mysterious properties of atomic nuclei to those of the neutron contained in them.

Supposing that the similar situation is realized at present, I proposed a compound hypothesis for new unstable particles to account for Nishijima-Gell-Mann's rule. In our model, the new particles are considered to be composed of four kinds of fundamental particles in the true sense, that is, nucleon, antinucleon, N^0 and anti- N^0 . If we assume that N^0 has such intrinsic properties as were assigned by Nishijima and Gell-Mann, we can easily get their even-odd rule for the composite particles as the result of the addition laws for the ordinary spin, the isotopic spin and the strangeness. In the next table, the models and the properties of the new particles are shown together with those of the fundamental particles in the true sense.

* The content of this letter was read before the annual meeting of the Japanese Physical Society held in October 1955.

A note on the same subject has also been published in Bulletin de L'Académie Polonaise des Sciences (Cl. III-vol. IV, No. 6, 1956)

Name	Model	Isotopic Spin	Strangeness	Ordinary Spin
\mathfrak{N}		1/2	0	1/2
$\bar{\mathfrak{N}}$		1/2	0	1/2
Λ		0	-1	1/2?
$\bar{\Lambda}$		0	1	1/2?
π	$\mathfrak{N} + \bar{\mathfrak{N}}$	1	0	0
$\theta(\tau)$	$\mathfrak{N} + \bar{\Lambda}$	1/2	1	0?
$\bar{\theta}(\bar{\tau})$	$\bar{\mathfrak{N}} + \Lambda$	1/2	-1	0?
Σ	$\mathfrak{N} + \bar{\mathfrak{N}} + \Lambda$	1	-1	1/2?
Ξ	$\bar{\mathfrak{N}} + \Lambda + \Lambda$	1/2	-2	1/2?

Here \mathfrak{N} and $\bar{\mathfrak{N}}$ denote nucleon and antinucleon respectively, whereas Λ and $\bar{\Lambda}$ denote Λ^0 and anti- Λ^0 respectively³⁾.

The asymmetric treatment of the Λ, Σ and Ξ particles, which experimentally were so similar, was hardly credible.

Four years later Ikeda, Ogawa and Ohnuki studied the symmetry of the Sakata model in the limit where proton, neutron and lambda all had equal masses¹⁾. One of their tables was unforgettable:

715

Progress of Theoretical Physics, Vol. 22, No. 5, November 1959

A Possible Symmetry in Sakata's Model for Bosons-Baryons System

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(Received July 28, 1959)

In this paper we study a possible symmetry in Sakata's model for the strongly interacting particles. In the limiting case in which the basic particles, proton, p , neutron, n and Λ -particle, Λ , have an equal mass, our theory holds the invariance under the exchange of p and Λ or n and Λ in addition to the usual charge independence and the conservation of electrical and hyperonic charge.

From our theory the following are obtained: (a) iso-singlet π_0' -meson state, which is a pseudo-scalar, exists, (b) the spin of Ξ -particle may be $(3/2)^+$ and (c) several resonating states in K - and π -nucleon scattering are anticipated to exist.

Table II

Class I	$M=6, M=8$	Note
$S=-1, I=1/2$	$-(A\bar{n})$ $(A\bar{p})$	\bar{K}^0 \bar{K}^-
$S=0, I=0$	$(p\bar{p}+n\bar{n}-2\Lambda\bar{\Lambda})/\sqrt{6}$	π^0
$S=0, I=1$	$(p\bar{n})$ $(p\bar{p}-n\bar{n})/\sqrt{2}$ $(n\bar{p})$	π^+ π^0 π^-
$S=1, I=1/2$	$(p\bar{\Lambda})$ $(n\bar{\Lambda})$	K^+ K^0

When I first read this paper in 1961, I found all known pseudoscalar mesons present and accounted for. The π^0 (η), whose existence was predicted here²⁾, had recently been discovered^{3,4)}. Clearly there was something important to be learned from this paper even though the classification of baryons seemed worse than ever (they were now systematically getting what appeared to be the wrong baryon states). Their striking prediction of the existence of the η was not mentioned by the experimental groups that discovered and studied this key particle.

In further elaboration of the Sakata model, Ikeda, Ogawa and Ohnuki gave a spectrum of what are now called "exotics"⁵⁾:

1073

Progress of Theoretical Physics, Vol. 23, No. 6, June 1960

A Possible Symmetry in Sakata's Model for Bosons-Baryons System. II

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(Received February 8, 1960)

In the previous paper we have discussed a possible symmetry among the proton, neutron and Λ -particle in Sakata's model and obtained some physically interesting results in bosons-baryons system. This symmetry is equivalent to the invariance of the theory under transformations of the unitary group $U(3)$ of degree three. We shall study a mathematical structure of our work in more detail.

Table V. The non-vanishing components and the expression of $v_{s_0 i_0}$, the values of (n_B, s_0, i_0) , m and m' , and the degree d of the representations

		$v_{s_0 i_0}$	n_B, s_0, i_0	m	m'	d
T^*	T^1	p	$1, 0, \frac{1}{2}$	$3/2$	$5/2$	3
T_λ	T_3	$\bar{\lambda}$	$-1, 1, 0$	$3/2$	$-5/2$	3
$T_\lambda^{(0)}$	T_3^1	$\bar{\lambda}p$	$0, 1, \frac{1}{2}$	3	0	8
$T_{(\lambda_1 \lambda_2)}^{(0) (s_1 s_2)}$	T_{33}^{11}	$\bar{\lambda}\bar{\lambda}pp$	$0, 2, 1$	8	0	27
$T_{[\lambda_1 \lambda_2]}^{(0) (s_1 s_2)}$	$T_{23}^{11} = -T_{32}^{11}$	$[\bar{n}, \bar{\lambda}]pp/\sqrt{2}$	$0, 1, 3/2$	6	9	10
$T_{(\lambda_1 \lambda_2)}^{(0) (s_1 s_2)}$	$T_{33}^{12} = -T_{33}^{21}$	$\bar{\lambda}\bar{\lambda}[p, n]/\sqrt{2}$	$0, 2, 0$	6	-9	10
$T_\lambda^{(0) (s_1 s_2)}$	T_3^{11}	$\bar{\lambda}pp$	$1, 1, 1$	$11/2$	$17/2$	15
$T_\lambda^{(0) (s_1 s_2)}$	$T_3^{12} = -T_3^{21}$	$\bar{\lambda}[p, n]/\sqrt{2}$	$1, 1, 0$	$7/2$	$-\frac{1}{2}$	6
$T_{(\lambda_1 \lambda_2)}^{(0) (s_1 s_2 s_3)}$	T_{33}^{111}	$\bar{\lambda}\bar{\lambda}ppp$	$1, 2, 3/2$	$23/2$	$35/2$	42
$T_{[\lambda_1 \lambda_2]}^{(0) (s_1 s_2 s_3)}$	$T_{23}^{111} = -T_{32}^{111}$	$[\bar{n}, \bar{\lambda}]ppp/\sqrt{2}$	$1, 1, 2$	$19/2$	$53/2$	15
$T_{(\lambda_1 \lambda_2)}^{(0) (s_1 s_2 s_3)}$	$T_{33}^{121} = -T_{33}^{211}$	$\bar{\lambda}\bar{\lambda}[p, n]p/\sqrt{2}$	$1, 2, \frac{1}{2}$	$17/2$	$-\frac{1}{2}$	24
$T_{(\lambda_1 \lambda_2)}^{(0) (s_1 s_2 s_3)}$	$T_{33}^{112} = -T_{33}^{211}$	$\bar{\lambda}\bar{\lambda}(ppn - npp)/\sqrt{2}$	$1, 2, \frac{1}{2}$	$17/2$	$-\frac{1}{2}$	24

This missed the point.

Murray Gell-Mann realized that a baryon classification problem existed and dealt with it in the beginning of 1961 in two different versions of a Caltech Synchrotron Report:

Report CTSL-20

CALIFORNIA INSTITUTE OF TECHNOLOGY

Synchrotron Laboratory
Pasadena, California

THE EIGHTFOLD WAY:
A THEORY OF STRONG INTERACTION SYMMETRY*

Murray Gell-Mann

March 15, 1961

(Second printing: April, 1962)

(Third printing: October, 1963)

(Preliminary version circulated Jan. 20, 1961)

*Research supported in part by the U. S. Atomic Energy Commission Contract No. AT(11-1)-68, and the Alfred P. Sloan Foundation.

This material was extensively revised before it was finally published as a relatively minor section of a Physical Review paper.

Both Synchrotron Reports contain a table which associates baryons with composite systems of leptons and bosons carrying baryon number 1:

Using the symbol \sim for "transforms

like", we define

$$\begin{aligned}
 \Sigma^+ &\sim \frac{1}{2} \bar{L}(\lambda_1 - i\lambda_2)l && \sim D^+_{\nu} \\
 \Sigma^- &\sim \frac{1}{2} \bar{L}(\lambda_1 + i\lambda_2)l && \sim D^0_{e^-} \\
 \Sigma^0 &\sim \frac{1}{\sqrt{2}} \bar{L} \lambda_3 l && \sim \frac{D^0_{\nu} - D^+_{e^-}}{\sqrt{2}} \\
 p &\sim \frac{1}{2} \bar{L}(\lambda_4 - i\lambda_5)l && \sim S^+_{\nu} \\
 n &\sim \frac{1}{2} \bar{L}(\lambda_6 - i\lambda_7)l && \sim S^+_{e^-} \\
 \Xi^0 &\sim \frac{1}{2} \bar{L}(\lambda_6 + i\lambda_7)l && \sim D^+_{\mu^-} \\
 \Xi^- &\sim \frac{1}{2} \bar{L}(\lambda_4 + i\lambda_5)l && \sim D^0_{\mu^-} \\
 \Lambda &\sim \frac{1}{\sqrt{2}} \bar{L} \lambda_8 l && \sim (D^0_{\nu} + D^+_{e^-} - 2S^+_{\mu^-})/\sqrt{6} \quad . \quad (3.5)
 \end{aligned}$$

The most graphic description of what we are doing is given in the last column, where we have introduced the notation D^0 , D^+ , and S^+ for the \bar{L} particles analogous to the \bar{l} particles $\bar{\nu}$, e^+ , and μ^+ respectively. D stands for doublet and S for singlet with respect to isotopic spin. Using the last column, it is easy to see that the isotopic spins, electric charges, and hypercharges of the multiplets are exactly as we are accustomed to think of them for the baryons listed.

The first version of this section on composite states transforming like baryons concludes with "what physical significance do we attach to the \bar{L} and ℓ when we say that the eight baryons N_i transform like $\frac{L\lambda, \ell}{\sqrt{6}}$ with respect to unitary spin? Certainly we are not claiming that the baryons must be bound states of leptons ℓ and heavy bosons \bar{L} under the influence of some very strong interaction. The leptons show no signs of having any strong couplings, and the heavy bosons \bar{L} , carrying baryon number 1 and lepton number -1, have made no appearance. For the time being, we cannot be sure of the physical significance of the analogy we have drawn, except possibly insofar as it concerns the weak interactions (see Section VI). But the physical consequences for the baryons and mesons of assuming the eight representations of unitary spin for the baryons are clear and precise."

In the second version this is replaced with "We shall attach no physical significance to the ℓ and \bar{L} 'particles' out of which we have constructed the baryons. The discussion up to this point is really just a mathematical introduction to the properties of unitary spin."

In the Physical Review paper the table of baryons is not given; the "abstract approach" is adopted:

PHYSICAL REVIEW

VOLUME 125, NUMBER 3

FEBRUARY 1, 1962

Symmetries of Baryons and Mesons*

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(Received March 27, 1961; revised manuscript received September 20, 1961)

The system of strongly interacting particles is discussed, with electromagnetism, weak interactions, and gravitation considered as perturbations. The electric current j_a , the weak current J_a , and the gravitational tensor θ_{ab} are all well-defined operators, with finite matrix elements obeying dispersion relations. To the extent that the dispersion relations for matrix elements of these operators between the vacuum and other states are highly convergent and dominated by contributions from intermediate one-meson states, we have relations like the Goldberger-Treiman formula and universality principles like that of Sakurai according to which the ρ meson is coupled approximately to the isotopic spin. Homogeneous linear dispersion relations, even without subtractions, do not suffice to fix the scale of these matrix elements; in particular, for the nonconserved currents, the renormalization factors cannot be calculated, and the universality of strength of the weak interactions is undefined. More information than just the dispersion relations must be supplied, for example, by field-theoretic models; we consider, in fact, the equal-time commutation relations of the various parts of j_a and J_a . These nonlinear relations define an algebraic system (or a group) that underlies the structure of baryons and mesons. It is suggested that the group is in fact $U(3) \times U(3)$, exemplified by the symmetrical Sakata model. The Hamiltonian density θ_{44} is not completely invariant under the group; the noninvariant part transforms according to a particular

representation of the group; it is possible that this information also is given correctly by the symmetrical Sakata model. Various exact relations among form factors follow from the algebraic structure. In addition, it may be worthwhile to consider the approximate situation in which the strangeness-changing vector currents are conserved and the Hamiltonian is invariant under $U(3)$; we refer to this limiting case as "unitary symmetry." In the limit, the baryons and mesons form degenerate supermultiplets, which break up into isotopic multiplets when the symmetry-breaking term in the Hamiltonian is "turned on." The mesons are expected to form unitary singlets and octets; each octet breaks up into a triplet, a singlet, and a pair of strange doublets. The known pseudoscalar and vector mesons fit this pattern if there exists also an isotopic singlet pseudoscalar meson χ^0 . If we consider unitary symmetry in the abstract rather than in connection with a field theory, then we find, as an attractive alternative to the Sakata model, the scheme of Ne'eman and Gell-Mann, which we call the "eightfold way"; the baryons N , Λ , Σ , and Ξ form an octet, like the vector and pseudoscalar meson octets, in the limit of unitary symmetry. Although the violations of unitary symmetry must be quite large, there is some hope of relating certain violations to others. As an example of the methods advocated, we present a rough calculation of the rate of $K^+ \rightarrow \mu^+ + \nu$ in terms of that of $\pi^+ \rightarrow \mu^+ + \nu$.

VIII. THE "EIGHTFOLD WAY"

Unitary symmetry may be applied to the baryons in a more appealing way if we abandon the connection with the symmetrical Sakata model and treat unitary symmetry in the abstract. (An abstract approach is, of course, required if there are no "elementary" baryons and mesons.) Of all the groups that could be generated by the vector weak currents, $SU(3)$ is still the smallest and the one that most naturally gives rise to the rules $|\Delta I| = \frac{1}{2}$ and $\Delta S/\Delta Q = 0, +1$.

There is no longer any reason for the baryons to belong to the $\mathbf{3}$ representation or the other spinor representations of the group $SU(3)$; the various irreducible spinor representations are those obtained by reducing direct products like $\mathbf{3} \times \mathbf{3} \times \mathbf{3}^*$, $\mathbf{3} \times \mathbf{3} \times \mathbf{3} \times \mathbf{3}^* \times \mathbf{3}^*$, etc.

Instead, the baryons may belong, like the mesons, to representations such as $\mathbf{8}$ or $\mathbf{1}$ obtained by reducing the direct products of equal numbers of $\mathbf{3}$'s and $\mathbf{3}^*$'s. It is then natural to assign the stable and metastable baryons N , Λ , Σ , and Ξ to an octet, degenerate in the limit of unitary symmetry.

Here the Sakata model is only of historical interest. It suggested $SU(3)$ as an approximate symmetry of the strong interactions, an approximate symmetry that was now defined by exact equal-time commutation relations. Compositeness of hadrons was no longer the issue.

Yuval Ne'eman viewed baryons similarly⁶⁾:

8.B

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DERIVATION OF STRONG INTERACTIONS FROM A GAUGE INVARIANCE

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Received 13 February 1961

Abstract: A representation for the baryons and bosons is suggested, based on the Lie algebra of the 3-dimensional traceless matrices. This enables us to generate the strong interactions from a gauge invariance principle, involving 8 vector bosons. Some connections with the electromagnetic and weak interactions are further discussed.

This was the first resolution of the baryon problem. The second was to come with the discovery of quarks.

Recognizing the approximate symmetry of the strong interactions had not been easy. For example, Gell-Mann⁷⁾ had previously developed an entirely different point of view related to earlier work of Schwinger⁸⁾ and Pais⁹⁾ in which the coupling $g_{\Lambda K}^2/4\pi$ was much less than $g_{N\pi}^2/4\pi$. Articles based on this "global symmetry" were still being published as late as the middle of 1961:

PHYSICAL REVIEW

VOLUME 122, NUMBER 6

JUNE 15, 1961

Some Considerations on Global Symmetry

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(Received February 3, 1961)

If the recently discovered Y^* state is related to the $T = \frac{1}{2}, J = \frac{3}{2}$ resonance in πp scattering, global symmetry considerations should become relevant. In this paper, global symmetry is discussed with a view to understanding its group structure. Also discussed is a possibility of reconciling the conflict, pointed out by Pais, between certain experimental results and global symmetry. The partial widths of the Y^* state are calculated and also those of the companion excited states Z^* and Ξ^* . A generalization of the quantum number G is discussed.

Almost one year after the "Eightfold Way" had been widely circulated, the simple groups C_2 (two-dimensional symplectic group), B_2 (five-dimensional orthogonal group), and the exceptional group G_2 were still considered by some as contenders for the symmetry of strong interactions:

REVIEWS OF MODERN PHYSICS

VOLUME 34, NUMBER 1

JANUARY, 1962

Simple Groups and Strong Interaction Symmetries*

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CONTENTS

Introduction		VI. Applications	28
I. Symmetries of the Lagrangian	3	A. Introductory Remarks	28
II. Lie Algebras of Simple Groups	4	B. Analysis of Invariant Amplitudes	29
III. Representations of Lie Algebras	7	C. Resonances and Mesons	31
A. General Properties of Representations	7	D. Model Built on SU_3	32
B. Characters of Representations of SU_3	9	E. Model Built on C_2	34
C. Characters of Representations of G_2	9	F. Model Built on B_2	36
D. Characters of Representations of C_2	10	G. Model Built on G_2	38
E. Synthesis of Representations of Lie Algebras	10		
F. Matrix Representations of SU_3	13		
G. Matrix Representations of C_2	15		
H. Matrix Representations of G_2	16		
IV. Composition and Decomposition of Lie Algebra Representations	16		
A. Geometric Characterization of a Representation	17		
B. Algebra of Sets of Points	17		
C. Construction of Weights and Multiplicities of Irreducible Representations	18		
D. Reduction of Direct Products of Representations	19		
V. Tensor Analysis of Simple Lie Groups	20		
A. Groups SU_m	20		
B. Group SU_3	22		
C. Group $C_2(B_2)$	24		
D. Group G_2	26		

INTRODUCTION

ONE of the most natural questions when one looks at the mass of uncorrelated data on elementary particle interactions¹ is whether a systematic pattern is emerging from this complexity. The penetration of controlled laboratory experiments into the multi-Bev energy region can only make such a question more acute. Several attempts² have already been made to unfurl the underlying symmetry of strong interactions, such as might exist above and beyond those symmetries, e.g., isotopic symmetry,³ which have already survived experimental tests.

In this article, we sharpen some tools which prove useful in formulating the consequences of proposed symmetries of a rather special type, namely, those symmetries which are characteristic of the simple Lie groups.

¹ See, for example, the *Proceedings of the Tenth Annual Conference on High Energy Nuclear Physics, Rochester, 1960, University of Rochester* (Interscience Publishers, Inc., New York, 1960).

² See, for example, B. d'Espagnat and J. Prentki, *Nuclear Phys.* **1**, 33 (1956); J. Schwinger, *Ann. Phys.* **2**, 407 (1957); M. Gell-Mann, *Phys. Rev.* **106**, 1296 (1957); A. Pais, *ibid.* **110**, 574 (1958); J. Tiomno, *Nuovo cimento* **6**, 69 (1957); R. E. Behrends, *ibid.* **11**, 424 (1959); D. C. Peaslee, *Phys. Rev.* **117**, 873 (1960); J. J. Sakurai, *ibid.* **115**, 1304 (1959).

³ See, for example, W. Heisenberg, *Z. Physik* **77**, 1 (1932); B. Cassen and E. U. Condon, *Phys. Rev.* **50**, 846 (1936); G. Breit, E. U. Condon, and R. D. Present, *ibid.* **50**, 825 (1936); G. Breit and E. Feenberg, *ibid.* **50**, 850 (1936).

* Supported in part by the National Science Foundation and by the Atomic Energy Commission.

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Even if SU(3) was accepted as an approximate symmetry of the strong interactions, the problem of assigning resonances to irreducible representations of this group was formidable. Errors were often made. The enormous symmetry breaking found in hadronic coupling constants led to a misclassification of the famous $N^*(3/2^+)$ and its partners:

IL NUOVO CIMENTO

VOL. XXV, N. 2

16 Luglio 1962

**The 27-fold Way and Other Ways:
Symmetries of Mesons-Baryon Resonances (*)**

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University of Chicago - Chicago, Ill.
Department of Physics, California Institute of Technology - Pasadena, Cal.*

(ricevuto il 2 Aprile 1962)

Summary. — Meson-baryon resonances are discussed within the framework of unitary symmetry based upon the Gell-Mann-Ne'eman baryon octet (the eightfold way). It is argued that the low-lying $J = \frac{3}{2}^+$ isobars ($N^*_\frac{1}{2}$, Y^*_0 , Y^*_1 , Y^*_2 , etc.) realize a twenty-seven-dimensional representation of SU(3). Purely group-theoretic considerations directly following from unitary symmetry are compared with dynamical considerations suggested by the coupling constant combinations required by unitary symmetry and R-symmetry. Higher resonances are briefly discussed.

Difficulties in understanding the manifestations of a partially conserved strangeness changing vector current led Nambu and Sakurai to argue that at least one particle should not even lie in an irreducible representation of SU(3):

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PHYSICAL REVIEW LETTERS

1 JULY 1963

κ MESON [K^* (725)] AND THE STRANGENESS-CHANGING CURRENTS OF UNITARY SYMMETRY*

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(Received 26 April 1963)

We wish to examine, within the framework of unitary symmetry, the hypothesis that the vector mesons M and \bar{M} (to be identified with the observed K^* meson of mass 885 MeV) are coupled to strangeness-changing currents that are conserved "as exactly as possible." It is pointed out that this hypothesis suggests the existence of $Y = \pm 1$, $T = 1/2$, and $J = 0^+$ mesons (with no unitary partners) whose couplings to other strongly inter-

acting particles vanish in the limit of exact unitary symmetry. The possible connection between the conjectured scalar meson and the experimentally observed κ meson (the K^* meson of 725 MeV) is discussed.

Inevitably, mathematical errors were also made. Oakes and Yang's argument that symmetry breaking mass formulae could not be used to classify hadrons was particularly confusing:

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MESON-BARYON RESONANCES AND THE MASS FORMULA*

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(Received 15 July 1963)

We have emphasized above some problems encountered in assigning the meson-baryon resonances to a pure multiplet in the octet symmetry scheme. In particular, we pointed out that the application of the mass formula to $N_{\frac{3}{2}}^*$, Y_1^* , $\Xi_{\frac{1}{2}}^*$ and Ω^- , regarded as forming a pure tenfold multiplet, is without theoretical justification. However, equally spaced energy levels are always empirically worthy of attention, and the search for the Ω^- should certainly be continued. We only emphasize that if the Ω^- is found and if it does satisfy the equal-spacing rule, it can hardly be interpreted as giving support to the octet symmetry model, at least not without the introduction of drastically new physical principles.

In dealing with these issues I argued that coupling constant relations, which previously had misguided many into adopting "global symmetry," should not be used for hadron classification unless these relations were valid in the presence of symmetry breaking¹⁰⁾. The Nambu-Sakurai paper was imaginative, but their assumptions were untested. I didn't understand the Oakes-Yang paper and hoped that the problems they raised would somehow go away. Eventually they did¹¹⁾.

The mainstream of high energy theoretical physics was concerned with other matters - dispersion theory, Regge poles and the "bootstrap," with the dominant philosophy of the day eloquently summarized by Chew and Frautschi:

PRINCIPLE OF EQUIVALENCE FOR ALL STRONGLY INTERACTING PARTICLES
WITHIN THE S-MATRIX FRAMEWORK*

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(Received October 30, 1961)

The notion, inherent in Lagrangian field theory, that certain particles are fundamental while others are complex, is becoming less and less palatable for baryons and mesons as the number of candidates for elementary status continues to increase. Sakata has proposed that only the neutron, proton, and Λ are elementary,¹ but this choice is rather arbitrary, and strong-interaction consequences of the Sakata model merely reflect the established symmetries. Heisenberg some years ago proposed an underlying spinor field that corresponds to no particular particle but which is supposed to generate all the observed particles on an equiva-

lent basis.² The spirit of this approach satisfies Feynman's criterion that the correct theory should not allow a decision as to which particles are elementary,³ but it has proved difficult to find a convincing mathematical framework in which to fit the fundamental spinor field. On the other hand, the analytically continued S matrix—with only those singularities required by unitarity⁴—has progressively, over the past half decade, appeared more and more promising as a basis for describing the strongly interacting particles. Our purpose here is to propose a formulation of the Feynman principle within the S-matrix framework.

The discovery of meson resonances was of interest because their existence had been predicted from a dispersion theoretical analysis of elastic electron-nucleon scattering data¹²⁾. Bootstrapping these resonances seemed possible:

SELF-CONSISTENT CALCULATION OF THE MASS AND WIDTH OF THE $J=1, T=1, \pi\pi$ RESONANCE

Fredrik Zachariasen*

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(Received June 28, 1961)

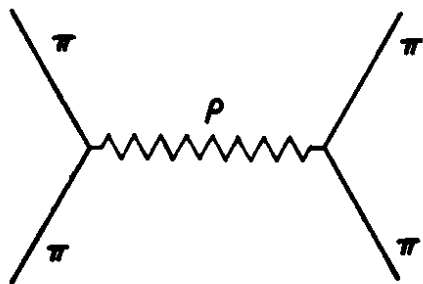


FIG. 1. The one ρ -meson exchange diagram.

The computation is explicit and straightforward, and an approximate numerical evaluation of the necessary two integrals yields

$$m_{\rho} \approx 950 \text{ Mev}, \quad \gamma_{\rho\pi\pi}^2/4\pi \approx 2.8.$$

There are no parameters to be adjusted in obtaining these results, other than the pion mass which only provides a dimension. The numbers are in fair agreement with the present experimental data,¹ which indicate something like

$$m_{\rho} \approx 750 \text{ Mev}, \quad \gamma_{\rho\pi\pi}^2/4\pi \approx 1.$$

There was hope that dynamics would determine symmetry and deviations from symmetry:

Origin of Internal Symmetries*

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(Received 1 May 1963)

Internal symmetries such as isotopic spin are not necessarily arbitrary constraints to be imposed at the beginning of a calculation. The bootstrap requirement that all particles be determined as composite states of one another leads naturally to symmetric solutions for masses and coupling constants.

I. INTRODUCTION

PHYSICAL systems are characterized by quantum numbers of energy, momentum, spin, and parity, whose origin is well understood; they arise from assumed symmetries of space-time. Some systems are also characterized by the internal quantum numbers of isotopic spin, hypercharge, and, more generally and less exactly, unitary spin, whose origins are less clear. We believe theory as initial data; rather they emerge from a self-consistent calculation. Let us anticipate that in a fully self-consistent universe there is room for a multiplicity of particles of the same species, that is, of the same spin and parity. The formal principles which instruct us how to determine the masses and couplings of particles of like species possess a symmetry with regard to these

that these quantum numbers and the associated symmetries are already implied by the bootstrap mechanism of *S*-matrix theory.¹ There is no need for additional principles either inside or outside quantum theory to explain them.

The fundamental point is this: The internal symmetries can be expressed as equalities among certain masses and among certain couplings. But the values of these masses and couplings are not inserted into the particles. That such symmetries lead to equality among the masses and interactions of particles of like species need not be regarded as a freak accident, but may well be the preferred possibility.

In the present paper this quite general notion will be explored only with regard to pion-nucleon interactions and isotopic symmetry.

Self-Consistent Deviations from Unitary Symmetry*

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(Received 24 June 1963)

A method of investigating the possible dynamic origin of symmetries among the strong interactions is illustrated by application to a model with vector mesons that are self-consistently bound states of one another. The SU_2 model, with eight vector mesons, is concentrated upon. All possible types of first-order perturbations are treated in the ladder approximation, and some second-order effects are also considered. The results emerging from a qualitative discussion uniquely suggest the possibility (in addition to the degenerate mass solution) of a self-supporting small mass splitting structure of the type leading to the Gell-Mann-Okubo mass formula. Moreover, SU_2 symmetry is necessarily retained, although the differentiation between charge and hypercharge is not possible in a theory which does not include electromagnetism.

The spectrum of particles and their pattern of coupling constants were to be self-consistently calculated. Constituents, other than the hadrons themselves, were out of the question.

All points of view were represented at Caltech. Feynman, Frautschi, Gell-Mann and Zachariasen were professors there, while Hung Cheng, Sidney Coleman, Roger Dashen, Bill Wagner and Ken Wilson were some of their more illustrious students. Glashow was a postdoctoral fellow.

I was a second-year graduate student at Caltech when Murray gave his first seminar on the "Eightfold Way." It was fantastic. Murray was really feeling his oats. As far as I was concerned, his classification of particles into irreducible representations of SU(3) was obviously correct, although the scientific community did not fully accept these assignments until the Ω^- was discovered three years later.

Murray's output in 1961 and 1962 was phenomenal, with one new paper appearing almost every two months:

PUBLICATIONS - Murray Gell-Mann

29. The Reaction $\gamma + \gamma \rightarrow \nu + \bar{\nu}$, Phys. Rev. Letters 6, 70 (1961).
30. Broken Symmetries and Bare Coupling Constants (with Fredrik Zachariasen), Phys. Rev. 123, 1065 (1961).
31. Form Factors and Vector Mesons (with F. Zachariasen), Phys. Rev. 124, 953 (1961).
32. Gauge Theories of Vector Particles (with Sheldon L. Glashow), Ann. Phys. 15, 437 (1961).
33. Symmetry Properties of Fields, Proceedings of Solvay Congress (1961).
34. Symmetries of Baryons and Mesons, Phys. Rev. 125, 1067 (1962).
35. Experimental Consequences of the Hypothesis of Regge Poles (with S. C. Frautschi and F. Zachariasen), Phys. Rev. 126, 2204 (1962).
36. Decay Rates of Neutral Mesons (with D. Sharp and W. G. Wagner), Phys. Rev. Letters 8, 261 (1962).
37. Factorization of Coupling to Regge Poles, Phys. Rev. Letters 8, 263 (1962).
38. High Energy Nuclear Scattering and Regge Poles (with B. M. Udgaonkar), Phys. Rev. Letters 8, 346 (1962).
39. Elementary Particles of Conventional Field Theory as Regge Poles (with M. L. Goldberger), Phys. Rev. Letters 9, 275 (1962).

All this was being worked out down the hall. What an example!

Richard Feynman also exerted his influence, both through his work and outlook. Solutions to problems were invariably based on simple ideas. Physical insight balanced calculational skill. And work was to be published only when it was correct, important, and fully understood. This was a stern conscience who practiced what he preached.

My first research project at Caltech was in experimental particle physics, where I worked closely with Alvin Tollestrup and Ricardo Gomez. Alvin was organizing a $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ experiment to be run at the Bevatron when I proposed using the same K^+ beam to look for a violation of time reversal symmetry in $K^+ \rightarrow \pi^0 + \mu^+ + \nu$ (an effect that was to be discovered four years later by Cronin and Fitch in $K_2^0 \rightarrow \pi^+ \pi^-$). After two and a half years of equipment building, the run at the Bevatron, and preliminary data analysis, I abandoned the experiment. The number of $K_{\mu 3}$ decays detected, much smaller than anticipated, did not justify a full-scale analysis.

Now at the beginning of my fourth year in the Fall of 1962 I finally decided to write a theoretical thesis. While previously working on the time reversal experiment I would occasionally talk to Murray about theoretical physics problems. Before he took a leave of absence to visit MIT in the Winter of 1962-63 he suggested that I see Feynman, who later became my thesis advisor. I did not see Murray again until I returned from CERN almost two years later.

For me, the origin of the quark model lay in the experiments that established the existence and properties of the ϕ meson:

EXISTENCE AND PROPERTIES OF THE ϕ MESON*

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Syracuse University, Syracuse, New York

(Received 27 March 1963)

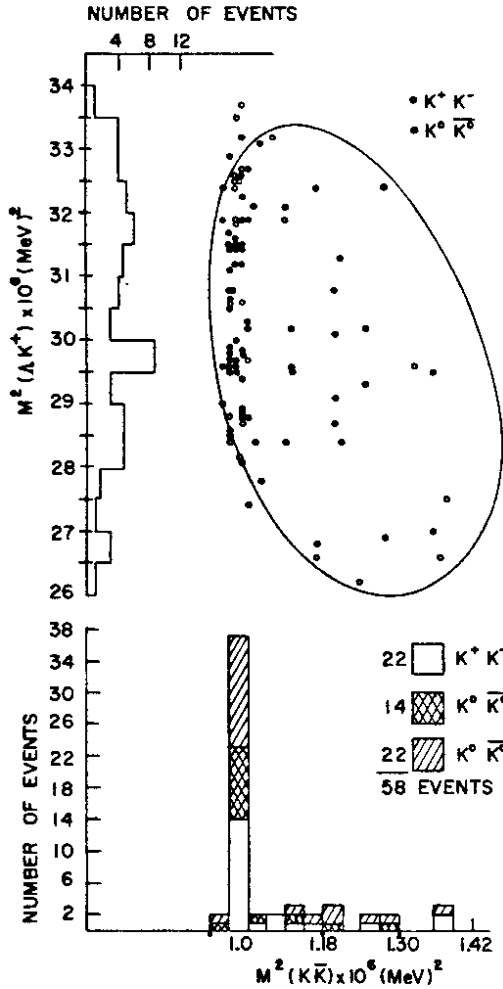


FIG. 1. Dalitz plot for the reaction $K^- + p \rightarrow \Lambda + K + \bar{K}$. The effective-mass distribution for $K\bar{K}$ and for ΛK^+ are projected on the abscissa and ordinate (see reference 7).

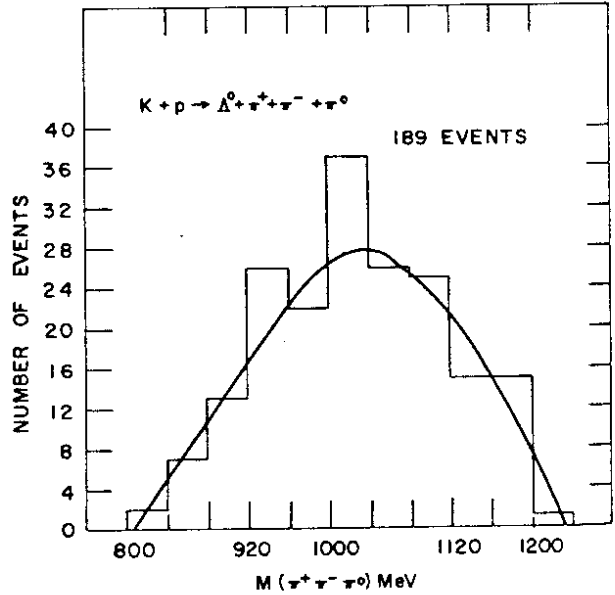


FIG. 4. The $M(\pi^+\pi^-\pi^0)$ distribution from the reaction $K^- + p \rightarrow \Lambda + \pi^+ + \pi^- + \pi^0$ after removing Y_1^* production events (see text).

$$\beta = \frac{\varphi \rightarrow \rho + \pi}{\varphi \rightarrow K + \bar{K}} \approx 0.35 \pm 0.2.$$

One can estimate β_J either from the ratio of phase space, barrier penetration, spin, and isospin factors which give $\beta_{J=1} \approx 4$ for an interaction radius of $(2m_\pi)^{-1}$ or from a dynamical approach as done by Sakurai² giving $\beta_{J=1} \approx 3$. The observed rate is lower than these predicted values by one order of magnitude; however the above estimates are uncertain¹⁸ by at least this amount so that this discrepancy need not be disconcerting.

The fact that the ϕ decayed predominantly into $K\bar{K}$ and not $\rho\pi$ was totally unintelligible despite the authors assurances that this suppression "need not be disconcerting." A spin one ϕ would decay into $\rho\pi$ or $K\bar{K}$ in a P-wave. Since the ϕ was just slightly above $K\bar{K}$ threshold, the P-wave $K\bar{K}$ mode was greatly suppressed. My estimate indicated that the $\rho\pi$ decay mode was at least two orders of magnitude below what might be naively expected. Feynman taught that in strong interaction physics everything that possibly can happen does, and with maximum strength. Only conservation laws suppress reactions. Here was a reaction that did not go when it should have. Many years later history was to repeat itself with the discovery of the $\psi(J)$.

To understand ϕ decay, I went back to the Sakata model. The work of Ikeda, Ogawa and Ohnuki gave the pseudoscalar mesons in terms of p, n and Λ . I assumed that the vector mesons were the same as the pseudoscalars, with two exceptions. First, the spin of the constituents added to one rather than zero. Second, the symmetry breaking, which was introduced through a Λ -nucleon mass splitting, mixed the isoscalar vector mesons in such a way as to separate the two distinguishable quantities $\Lambda\bar{\Lambda}$ and $(\bar{p}p + \bar{n}n)$ from each other,

$$\begin{aligned}\phi &\sim \bar{\Lambda}\Lambda \\ \omega &\sim (\bar{p}p + \bar{n}n)/\sqrt{2}\end{aligned}$$

This last point was tricky because the isoscalar pseudoscalar mesons did not mix strongly.

Justification for this vector meson mixing came by assuming that the mass differences of vector mesons equalled the mass differences of their constituents. This gave mass formulae that worked:

$$\begin{aligned}m^2(\omega) &\approx m^2(\rho), \\ (784)^2 &\quad (750)^2 \\ m^2(\phi) &\approx 2 m^2(K^*) - m^2(\rho), \\ (1018)^2 &\quad (1007)^2\end{aligned}$$

and with a little fiddling even

$$(m^2(\omega) - m^2(\rho))/2 \approx m^2(\phi) + m^2(\rho) - 2 m^2(K^*),$$

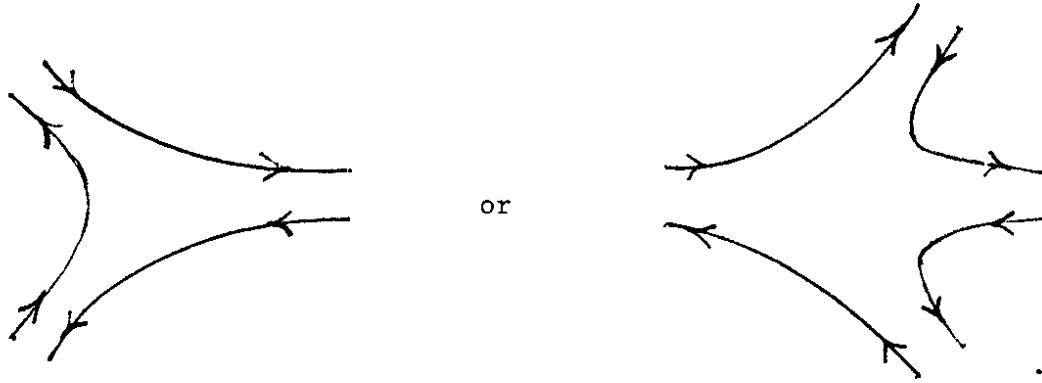
which was correct to the known accuracy of the masses. The fact that

$$m^2(K^*) - m^2(\rho) \approx m^2(K) - m^2(\pi)$$

was also suggestive. Understanding why the pseudoscalars mixed weakly would come later ¹⁰⁾.

A theory of hadronic decays within the framework of the Sakata model did not exist; it was now required. I supposed that a meson decays when its two

constituents separate. Since they are tightly bound, their separation must lead to the creation of pairs. In the notation that was developed shortly thereafter,



Since the ϕ was made of Λ and $\bar{\Lambda}$, constituents that did not appear in ρ or π , $\phi \rightarrow \rho\pi$ could not occur. This argument convinced me that hadrons had constituents!

Okubo was also puzzled by ϕ decay. He obtained mass formulae and the suppression of $\phi \rightarrow \rho\pi$ with the ansatz that the trace of a certain matrix should not explicitly appear in any mathematical expression:

ϕ -MESON AND UNITARY SYMMETRY MODEL [†]

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Received 13 May 1963

In the unitary symmetry model, the vector octet ($\rho, \omega, K^*, \bar{K}^*$) can be specified ⁴⁾ by a traceless tensor F_ν^μ . However, as we have noted, we have to take into account of the ninth boson ϕ together with the octet. This means that we are dealing with a nonet rather than an octet, and a singlet. We can always combine both into a reducible non-traceless tensor G_ν^μ by setting

$$G_\nu^\mu = F_\nu^\mu + \frac{1}{\sqrt{3}} \delta_\nu^\mu \phi. \quad (11)$$

We propose that the ϕ meson should appear always in this combination together with F_ν^μ , but should not appear singly without being accompanied by its

counter part F_ν^μ . When we note $G_\lambda^\lambda = \sqrt{3}\phi$, this means that we are requiring non-appearance of G_λ^λ explicitly in all mathematical expressions. Unfortunately, the present author could not justify this "ansatz" on a more satisfactory mathematical ground, but it may not be impossible that such an assumption could hold if the interaction Hamiltonians are restrictive with some new kinds of symmetries. Only reason for presenting this scheme in this note is simply to show a possible existence of an unified theory which is capable to explain many experimental results in a systematical way.

Then, the eigen-masses of $\tilde{\omega}$ and $\tilde{\phi}$ can be shown to satisfy the following relations:

$$m(\tilde{\omega}) = m(\rho), \quad (16)$$

$$[m(\tilde{\phi})]^2 = 2[m(K^*)]^2 - [m(\rho)]^2. \quad (17)$$

One thing interesting in our scheme is that in the lowest order with respect to H_I , we have

$$\Gamma(\tilde{\phi} - \rho + \pi) = 0. \quad (19)$$

Finally, we give expressions for $\rho, K^*, \tilde{\omega}$ and $\tilde{\phi}$ in terms of G_ν^μ :

$$\rho_+ = G_1^2, \rho_- = G_2^1, \rho_0 = \frac{1}{\sqrt{2}}(G_1^1 - G_2^2), \tilde{\omega} = \frac{1}{\sqrt{2}}(G_1^1 + G_2^2), \quad (21)$$

$$K_+^* = G_1^3, K_0^* = G_2^3, K_-^* = G_3^1, K_0^* = G_3^2, \tilde{\phi} = -G_3^3.$$

The connection with constituents was absent.

The baryon problem was now revived. I worked compulsively trying to solve it. One day while I was looking at the review paper of Behrends, Dreitlein, Fronsdaal and Lee I noticed their multiplication table of SU(3) representations:

STRONG INTERACTION SYMMETRIES

TABLE IV. Representations of SU_3 . All mixed tensors are supposed to be traceless, e.g., $\psi_{aa} = 0$. The missing representation "64" is $D^{64}(3,3)$ with the basis $\psi_{a_1 a_2 a_3}^{b_1 b_2 b_3}$ and the isotopic content $0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2, 2, 2, \frac{5}{2}, \frac{5}{2}, 3$. The dimension of $D(\lambda_1, \lambda_2)$ is $\frac{1}{2}(\lambda_1 + 1) \times (\lambda_2 + 1)(\lambda_1 + \lambda_2 + 2)$. The regular representation is $D^8(1,1)$.

Complete designation	Abbr. design	Highest weight	Fig. no.	Isotopic content	Basic	$\otimes D^8(1,0)$	$\otimes D^6(2,0)$	$\otimes D^8(1,1)$	$\otimes D^{10}(3,0)$
$D^3(0,0)$	1	(0,0)		0	ψ	3	6	8	10
$D^3(1,0)$	3	$\frac{1}{2}(\sqrt{3}, 1)$	2(a)	$0, \frac{1}{2}$	ψ_a	$6+3^*$	$10+8$	$15+6^*+3$	$15+15$
$D^3(0,1)$	3*	$\frac{1}{2}(\sqrt{3}, -1)$	2(b)	$0, \frac{1}{2}$	ψ^a	$8+1$	$15+3$	15^*+6+3^*	$24+6$
$D^6(2,0)$	6	$\frac{1}{2}(\sqrt{3}, 1)$	2(c)	$0, \frac{1}{2}, 1$	ψ_{ab}	$10+8$	$15'+15+6^*$	$24+15^*+6+3^*$	$24+21+15^*$
$D^6(0,2)$	6*	$\frac{1}{2}(\sqrt{3}, -1)$	2(d)	$0, \frac{1}{2}, 1$	ψ^{ab}	15^*+3^*	$27+8+1$	24^*+15+6^*+3	$42+15+3$
$D^8(1,1)$	8	$\frac{1}{2}(\sqrt{3}, 0)$	2(e)	$0, \frac{1}{2}, \frac{1}{2}, 1$	ψ_a^b, χ_A	$15+6^*+3$	$24+15^*+6+3^*$	$27+10+10^*+8+8+1$	$35+27+10+8$
$D^{10}(3,0)$	10	$\frac{1}{2}(\sqrt{3}, 1)$	22	$0, \frac{1}{2}, 1, \frac{3}{2}$	ψ_{abc}	$15'+15$	$24+21+15^*$	$35+27+10+8$	$35+28+27+10$
$D^{10}(0,3)$	10*	$\frac{1}{2}(\sqrt{3}, -1)$		$0, \frac{1}{2}, 1, \frac{3}{2}$	ψ^{abc}	24^*+6^*	$42^*+15^*+3^*$	$35^*+27+10^*+8$	$64+27+8+1$
$D^{15}(2,1)$	15	$\frac{1}{2}(\sqrt{3}, +\frac{1}{2})$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, \frac{3}{2}$	ψ_{bc}^a				
$D^{15}(1,2)$	15*	$\frac{1}{2}(\sqrt{3}, -\frac{1}{2})$							
$D^{15}(4,0)$	15'	$\frac{1}{2}(\sqrt{3}, +1)$							
$D^{15}(0,4)$	15''	$\frac{1}{2}(\sqrt{3}, -1)$		$0, \frac{1}{2}, 1, \frac{3}{2}, 2$	ψ_{abcd}				
$D^{21}(5,0)$	21	$\frac{1}{2}(\sqrt{3}, +1)$		$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{3}{2}$	ψ_{abcde}				
$D^{21}(0,5)$	21*	$\frac{1}{2}(\sqrt{3}, -1)$							
$D^{24}(3,1)$	24	$\frac{1}{2}(\sqrt{3}, +\frac{1}{2})$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, \frac{3}{2}, \frac{3}{2}, 2$	ψ_{bcde}^a				
$D^{24}(1,3)$	24*	$\frac{1}{2}(\sqrt{3}, -\frac{1}{2})$							
$D^{27}(2,2)$	27	$\frac{1}{2}(\sqrt{3}, 0)$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, 2$	ψ_{cde}^{ab}				
$D^{28}(6,0)$	28	$(\sqrt{3}, +1)$		$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{3}{2}, 3$	ψ_{abcdef}				
$D^{28}(0,6)$	28*	$(\sqrt{3}, -1)$							
$D^{35}(4,1)$	35	$\frac{1}{2}(\sqrt{3}, +3)$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, \frac{3}{2}, \frac{3}{2}, 2, \frac{3}{2}$	ψ_{bcde}^a				
$D^{35}(1,4)$	35*	$\frac{1}{2}(\sqrt{3}, -3)$							
$D^{36}(7,0)$	36	$\frac{1}{2}(\sqrt{3}, +1)$		$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{3}{2}, 3, \frac{3}{2}$	$\psi_{abcdefg}$				
$D^{36}(0,7)$	36*	$\frac{1}{2}(\sqrt{3}, -1)$							
$D^{42}(3,2)$	42	$\frac{1}{2}(\sqrt{3}, +1)$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, 2, \frac{3}{2}$	ψ_{cde}^{ab}				
$D^{42}(2,3)$	42*	$\frac{1}{2}(\sqrt{3}, -1)$							
$D^{45}(8,0)$	45	$\frac{1}{2}(\sqrt{3}, +1)$		$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{3}{2}, 3, \frac{3}{2}, 4$	$\psi_{abcdefgh}$				
$D^{45}(0,8)$	45*	$\frac{1}{2}(\sqrt{3}, -1)$							
$D^{48}(5,1)$	48	$(\sqrt{3}, +\frac{1}{2})$		$0, \frac{1}{2}, \frac{1}{2}, 1, 1, \frac{3}{2}, \frac{3}{2}, 2, \frac{3}{2}, 3$	ψ_{bcdef}^a				
$D^{48}(1,5)$	48*	$(\sqrt{3}, -\frac{1}{2})$							

Almost reflexively I started to work out the irreducible representations used to classify the baryons in the Sakata model. To decompose $3 \times 3 \times 3^*$, I took $D^3(1,0)$ from the first column and multiplied it by $D^3(1,0)$ from the first row to get $6 + 3^*$. Then taking $D^6(2,0)$ from the first column and multiplying it by $D^3(1,0)$ from the first row I got $10 + 8$, which I immediately recognized as the wrong answer. The product 6×3 had been formed instead of $6 \times 3^*$. There was no $D^3(0,1)$ in the first row. $D^3(0,1)$ had to be taken from the first column and $D^6(2,0)$ from the first row to obtain $15 + 3$. The lack of symmetry and the confusing notation of the Table had misled me into multiplying $3 \times 3 \times 3$ instead of $3 \times 3 \times 3^*$.

Although the 8 and 10 representations were not the correct ones in the Sakata model, they were almost certainly empirically correct. Therefore baryons had to be constructed from $3 \times 3 \times 3$, which meant the constituents had baryon number $1/3$. With the isotopic spin and strangeness assignments of the p, n and Λ , the Gell-Mann-Nishijima relation

$$Q = e \left(I_z + \frac{B+S}{2} \right)$$

then gave fractional charges to the constituents. Fractional charges bothered me because I wanted a correspondence between leptons and constituents of hadrons. To have one set of these particles integrally-charged and the other set fractionally-charged was ugly, but at this point there seemed to be no choice.

This was a fantastically exciting time. It was impossible to finish even the simplest calculation without jumping up, pacing back and forth for a few minutes, and rushing back to see if things were working after all.

Each constituent was represented by a regular polygon. Heavier constituents were drawn larger and had more vertices. There was one exception. Since pentagons were harder to draw than squares, and evidence for the fourth constituent had not yet been discovered, the series of constituents started, rather than ended, with a circle. The constituents were called "aces," in part because four leptons were known at that time. The total number of con-

stituents was, of course, unknown. The correspondence between the original and current notation is:

<u>Ace</u>	<u>Quark</u>
● p ₀	u
▲ m ₀	d
■ λ ₀	s
◆	c

The forces binding constituents into hadrons were represented by strings. For example, the eight baryons looked like:

$$\begin{aligned}
 p &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \triangle \quad \bullet \end{array} - \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \triangle \end{array} \right) & n &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \triangle \quad \bullet \end{array} - \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \bullet \quad \triangle \end{array} \right) \\
 \Lambda &= \frac{1}{\sqrt{12}} \left(\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \blacksquare \quad \triangle \end{array} - \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \blacksquare \quad \bullet \end{array} + \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \bullet \quad \blacksquare \end{array} - \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \triangle \quad \blacksquare \end{array} + 2 \begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \bullet \quad \triangle \end{array} - 2 \begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \triangle \quad \bullet \end{array} \right) \\
 \Sigma^0 &= \frac{1}{2} \left(\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \blacksquare \quad \triangle \end{array} + \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \blacksquare \quad \bullet \end{array} - \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \triangle \quad \blacksquare \end{array} - \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \bullet \quad \blacksquare \end{array} \right) \\
 \Sigma^+ &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \blacksquare \end{array} - \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \blacksquare \quad \bullet \end{array} \right) & \Sigma^- &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \blacksquare \quad \triangle \end{array} - \begin{array}{c} \triangle \\ \diagup \quad \diagdown \\ \triangle \quad \blacksquare \end{array} \right) \\
 \Xi^0 &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \bullet \quad \blacksquare \end{array} - \begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \blacksquare \quad \bullet \end{array} \right) & \Xi^- &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \blacksquare \quad \triangle \end{array} - \begin{array}{c} \blacksquare \\ \diagup \quad \diagdown \\ \triangle \quad \blacksquare \end{array} \right)
 \end{aligned}$$

Three aces connected by strings was called a Trey (the emblem for this conference?).

The coupling of hadrons to one another was determined by the number of ways the constituent aces could move from one hadron to another with the appropriate creation of pairs. For example, the coupling of ω to $K^+ K^-$ was represented by:

$$\begin{aligned}
 j\omega K^+ K^- &= \frac{1}{\sqrt{2}} \times \left[\begin{array}{c} \text{Diagram 1: } \lambda_0 \text{ and } \bar{\lambda}_0 \text{ paths} \\ \text{Diagram 2: } \lambda_0 \text{ and } \bar{\lambda}_0 \text{ paths} \end{array} \right] + \frac{1}{\sqrt{2}} \times \left[\begin{array}{c} \text{Diagram 3: } \lambda_0 \text{ and } \bar{\lambda}_0 \text{ paths} \\ \text{Diagram 4: } \lambda_0 \text{ and } \bar{\lambda}_0 \text{ paths} \end{array} \right] \\
 &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 = 1/\sqrt{2}
 \end{aligned}$$

In the early notation, this looked like: $\langle |\bar{\omega} K^+ K^-| \rangle =$

a. $\langle | \left[\left(\frac{1}{\sqrt{2}} \circ + \frac{1}{\sqrt{2}} \blacktriangle \right) \square \right] (\circ \blacksquare) | \rangle =$

b. $\frac{1}{\sqrt{2}} \langle | (\circ \square + \blacktriangle \square) (\circ \blacksquare) | \rangle =$

c. $\frac{1}{\sqrt{2}} \langle | \left[\begin{array}{c} \circ \blacksquare \\ \bullet \square \end{array} \right] + \blacktriangle \square \quad \circ \blacksquare \quad | \rangle =$

d. $\frac{1}{\sqrt{2}} \langle | \rangle + \frac{1}{\sqrt{2}} \langle | \blacktriangle \square \quad \circ \blacksquare \quad | \rangle =$

e. $\frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

The quantum numbers of the low-lying meson and baryon states were worked out and tentatively associated with observed enhancements in invariant mass distributions or resonances in phase shift analyses. Partial widths for the decays of these resonances were given. For example:

$\bar{A}A$ System	$\langle \vec{S} \cdot \vec{L} \rangle$	$C_{J^{PG}}$ for π like number	$m(\pi)$	$m(K)$	$m(\pi)$ or $m(\omega)$ $m(\phi)$
1S_0	0	$^+0^- -$	135	494	548
3S_1	0	$^-1^- +$	750	890	784 1019
3P_0	-2	$^+0^+ -$	550?	725	775?
1P_1	0	$^-1^+ +$	1140?	1232 ⁴⁶⁾	1260? or 1140? 1320?
3P_1	-1	$^+1^+ -$	1200 ⁴⁵⁾	1290?	1320? or 1200 1370?
3P_2	1	$^+2^+ -$	>1200?	$\sqrt{m^2(\pi)+0.22}$	$\sqrt{m^2(\pi)+0.29}$ or $m(\pi) \sqrt{m^2(\pi)+0.44}$
3D_1	-3	$^-1^- +$	1220	1320?	1220? 1410
1D_2	0	$^+2^- -$	>1200?	$\sqrt{m^2(\pi)+0.22}$	$\sqrt{m^2(\pi)+0.29}$ or $m(\pi) \sqrt{m^2(\pi)+0.44}$
3D_2	-1	$^-2^- +$	>1200?	"	" " "
3D_3	2	$^-3^- +$	>1200?	"	" " "

Table 7 We list here the low angular momentum systems that may be formed from an ace and an anti-ace. Certain resonances have been tentatively classified in this scheme. $\langle \bar{S} \cdot \bar{L} \rangle$ gives the expected value of the spin times the orbital angular momentum. It is tempting to conjecture that this is a pertinent quantity in ordering the energy levels of the $\bar{A}A$ system.

Decays of Octet	M_Y (MeV)	p (MeV)	Γ theory (MeV)	Γ exp. (MeV)
$N_Y \rightarrow \pi N$	1515 ± 3	452	80 (input)	80
$\Lambda_Y \rightarrow \bar{K}N$	1635?	376	25	< 40
$(\pi\Sigma)$		362	7.6	
$\Sigma_Y \rightarrow \pi\Lambda$	1660 ± 10	441	6.6	13
$(\bar{K}N)$		402	suppressed	< 2
$\pi\Sigma$		382	19.3	11
$\Xi_Y \rightarrow \pi\Xi$	1770 ± 25	375	suppressed	
$(\bar{K}\Lambda)$		342	1.6	
$(\bar{K}\Sigma)$		246	2.9	

Decays of Singlet	M_δ (MeV)	p (MeV)	Γ theory (MeV)	Γ exp. (MeV)
$\Lambda_\delta \rightarrow \pi\Sigma$	1519 ± 2	261	9 (input)	9
$\bar{K}N$		238	4.5	5

Table 4 $M_{\gamma,\delta}$, p, and Γ represent the mass of the decaying baryon, the final state momentum, and the width. Decay modes that have not yet been observed are included within parentheses in column 1. Although the $\pi\Xi$ decay channel of the Ξ_γ is suppressed by unitary symmetry, the large phase space available for this mode coupled with the breaking of the symmetry may account for the fact that $\Xi_\gamma \rightarrow \pi\Xi$ has been seen.

The rules for meson-baryon coupling restricted the f/d ratio to be one of several values. Of these, f/d = 1 was chosen for the $3/2^-$ and $5/2^+$ octets.

All this work is essentially correct. The assignment of states to the $3/2^-$ octet differed from that previously given:

EIGHTFOLD-WAY ASSIGNMENTS FOR $Y_1^*(1660)$ AND OTHER BARYONS†

Sheldon L. Glashow* and Arthur H. Rosenfeld

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 17 December 1962)

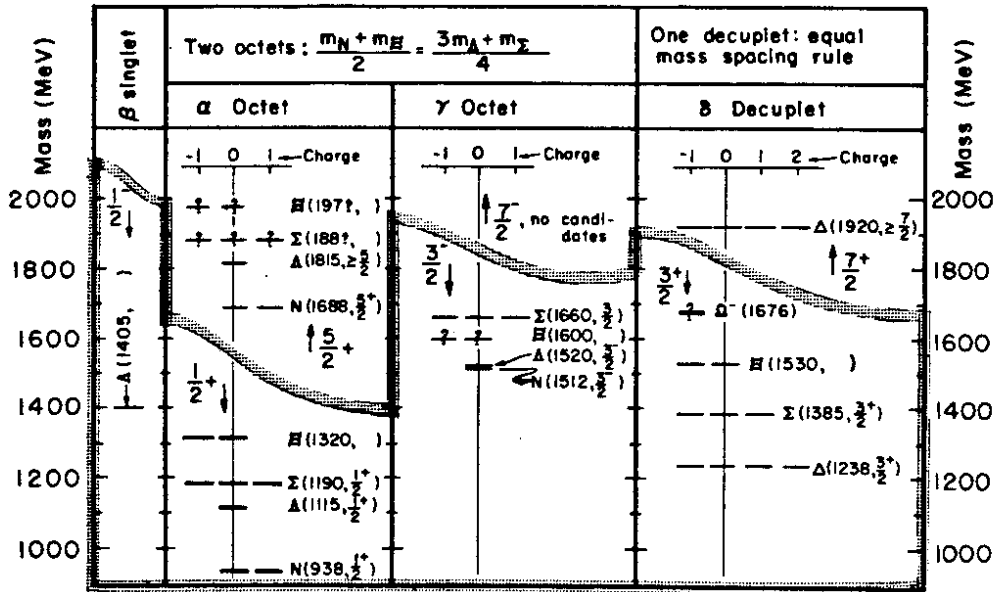


FIG. 1. Baryons: the four unitary multiplets and their Regge recurrences. Spin and parity assignments J^P are written beside each particle if they are supported by any experimental evidence; if not, J^P have been conjectured by assigning one known resonance to each set of quantum numbers. The notation was introduced in the Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), pp. 783 and 325. Observe that the families so defined coincide with the unitary multiplets of the eightfold way. Heavy bars show stable or metastable particles; light lines show resonances. States predicted by the eightfold way but not yet seen are indicated by question marks. The masses of Ξ_γ and Ω_δ^- follow from the mass formulas alone; those of the $\frac{5}{2}^+ \Sigma_\alpha$ and Ξ_α also require the assumption of nearly parallel Regge trajectories.

Two-body partial widths for the γ octet

Resonance and total width Γ	Decay mode	Momentum (MeV/c)	Width, Γ (MeV)	
			Experimental ^a	Calculated ^b
			γ octet	
$\Xi(1600?)$	$\Xi\pi$	220	?	0.6^c
$\Sigma(1600)$	$\bar{K}N$	406	3^d	3
$\Gamma=40$ MeV	$\Lambda\pi$	441	11	Input= 11
	$\Sigma\pi$	386	13	Input= 13
$\Lambda(1520)$	$\bar{K}N$	244	5	6
$\Gamma=16$ MeV	$\Sigma\pi$	267	9	Input= 8
$N(1512)$	$N\pi$	450	80	67
$\Gamma=100$ MeV				

With satisfaction and relief we find that the calculated results are completely compatible with experiment.

In the Glashow-Rosenfeld assignments, the nucleon-like member of the $3/2^-$ γ octet had a mass approximately equal to that of the lambda-like member, while the sigma-like member was heaviest of all. While this pattern of symmetry breaking was consistent with the Gell-Mann-Okubo mass formula, it was unlikely in the ace model where strange aces contributed more mass to hadrons than non-strange ones.

Particle classification was difficult because many peaks in invariant mass plots were spurious:

EVIDENCE FOR $\pi^+ \pi^-$ RESONANCES AT 395- AND 520-MeV EFFECTIVE MASS*

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and

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and

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(Received June 21, 1962)

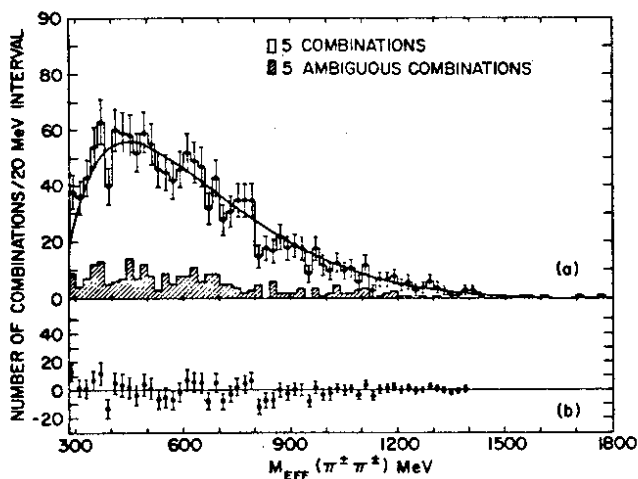


FIG. 2. (a) Histogram of the effective-mass distribution for the 1486 $(\pi^+ \pi^+ \pi^+)$ combinations from Reactions IIa, b and IIIa, b. The smooth curve is the invariant phase-space distribution normalized to the total number of events. (b) The distribution in (a) with the smooth curve subtracted. The errors shown are \sqrt{N} , where N is the total number of pion pairs per 20-MeV interval before subtraction.

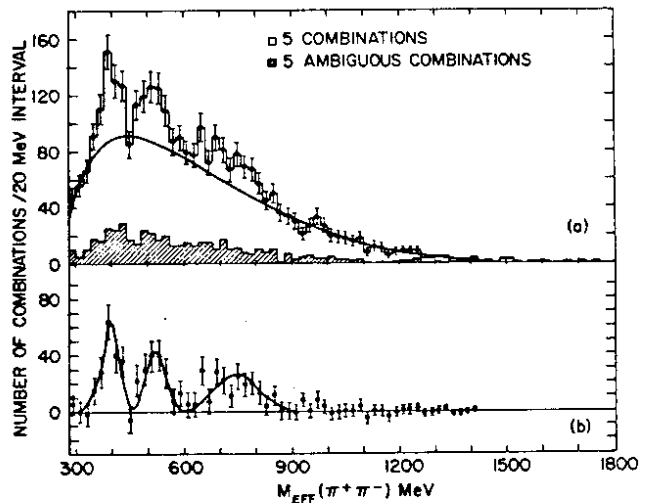


FIG. 3. (a) Histogram of the effective-mass distribution for the 2972 $(\pi^+ \pi^+ \pi^-)$ combinations from Reactions IIa, b and IIIa, b. The smooth curve is the invariant phase-space distribution normalized to the events with mass >850 MeV. (b) The distribution in (a) with the smooth curve subtracted. The errors shown are \sqrt{N} , where N is the total number of pion pairs per 20-MeV interval before subtraction.

The evidence for the $\pi^+\pi^-$ "resonances" looked, at least superficially, as impressive as the first evidence for the ϕ :

POSSIBLE RESONANCES IN THE $\Xi\pi$ AND $K\bar{K}$ SYSTEMS

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 R. R. Rau, N. P. Samios, I. O. Skillicorn,^{**} and S. S. Yamamoto
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and

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 Syracuse University, Syracuse, New York
 (Received July 2, 1962)

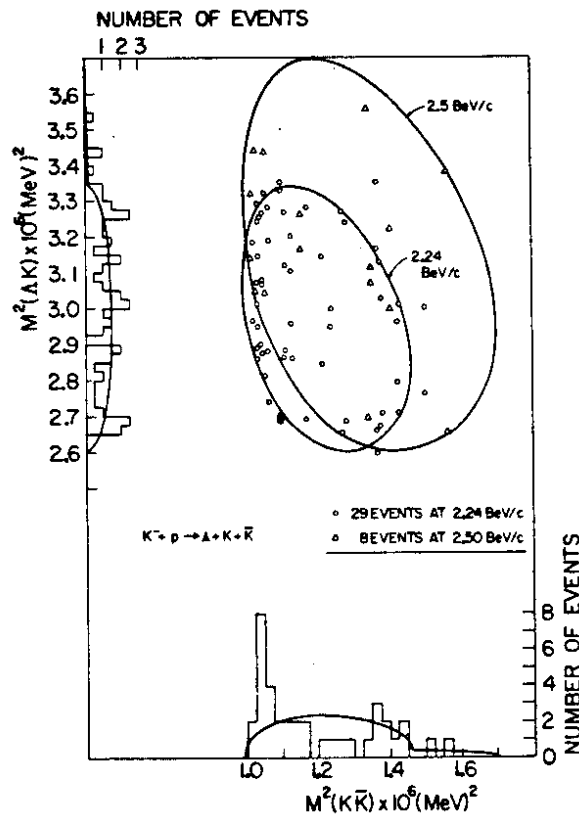


FIG. 2. The Dalitz plot for the channel $\Lambda K\bar{K}$ projected on the $M^2(K\bar{K})$ and $M^2(\Lambda K)$ axes. The solid curves on the projections are the invariant phase-space curves normalized to the total number of events.

Matts Roos published "Tables of Elementary Particles and Resonant States" in the Reviews of Modern Physics in April 1963. This was a definitive compilation. Later it would grow into the widely used "Review of Particle Properties" (affectionately referred to as the "Rosenfeld Tables"). I found a copy of Roos's 1963 paper in my notes, with this table of meson states:

TABLE IIb. Mesonic Resonant States, March 1963.

Symbol	Charge	Isospin	Spin	Parity G-parity	S	Mass		Full width Γ (MeV)	Life- time Γ^{-1} (1/m.e)	Production		Decay			References
						(MeV)	(m.e)			Process	k_{lab} (MeV)	Modes	Branching ratio (%)	Q (MeV)	
K_1^*	-	$\frac{1}{2}$			1	1630 ± 100	11.7			πp	3534	$(K^+\pi\pi)^-$ $(K^0\pi)^-$ $(K^-\pi)^-$ others		470 < 100 225	55
K_2^*	+				1							same, charge +		same	55
ω	0				0	1340 ± 70	9.6			πp	2287	$(\rho\pi\pi)^0$ others		290	55
ω_1	0				0	1275 ± 25	9.1			πp	2125	K^0K^0 K^+K^-		279 287	24
K^{*0}					1	1260	9.0			πN		$K(\pi\pi)$			55
f	0	0	2	++	0	1253 ± 20	9.0	100 ± 50	1.4	πp	2070	$\pi^+\pi^+$	100	970	33
K_1^*	--	$\frac{1}{2}$			1	1150 ± 50	8.2			πp	2250	$K^+\pi^+\pi^+$ $K^0\pi^+\pi^-$		373 373	55 55
χ_1	-	1			0	1050	7.5			πp	1620	$\pi^+\pi^+\pi^0$ $\pi^+\pi^-(\pi\pi)^0$		496 481-491	51 53
χ_2	0	0	even	++	0	1040 ± 40	7.4			K^-p	1780	K^0K^0 even number π 's		44	58
χ_1	0	0	odd	--	0	1020	7.3	< 3	> 47	K^-p	1760	K^0K^0 odd number π 's		24	59
ψ_2	-	2			0	990	7.2			πp	1490	$\pi^+\pi^-$	100	711	52
ψ_2	0				0	0						$\pi^+\pi^+$	100	711	52
ψ_2	++				0	0						$\pi^+\pi^+$	100	711	52
K_1^*	-	$\frac{1}{2}$	1	-	-1	888 ± 3	6.4	50 ± 10	2.8	K^-p	1074	$K^0\pi^-$ $K^-\pi^0$ $K^-\pi^+$	60 ± 16 40 ± 16	252 261 256	34
K_1^*	0				-1					K^-p	1078	$K^0\pi^+$ $K^+\pi^0$		257	34
K_1^*	+				1					πp	1834	$K^0\pi^+$ $K^+\pi^0$	67 33	252 261	34
K_1^*	0				1					πp	1657	$K^+\pi^0$ $K^+\pi^-$ $K^0\pi^0$		256 257	35, 34
ϕ_1	0				0	885 ± 10	6.3			πp	1284	$\pi^+\pi^-$		606	36
ω	0	0	1	--	0	781.1 ± 0.8	5.6	< 12	> 12	$\bar{p}p$		neutr. $\pi^+\pi^+\pi^0$ $\pi^+\pi^-\pi^0$ $\pi^0\pi^+\pi^-$ $\pi^0\pi^0\pi^0$ $\pi^+\pi^-\pi^+$ $\pi^+\pi^-\pi^-$	0.12 ± 0.03 < 2 < 12 < 5 4	373 503 232 223 503	37 53 53
ρ	-	1	1	-+	0	757 ± 5	5.4	120 ± 10	1.2	πp	1029	$\pi^+\pi^0$ $\pi^+\pi^+$ $\pi^0\pi^0$ $\pi^-\pi^0$ $\pi^-\pi^+$ $\pi^-\pi^-$ neutrals	$94 (+6/-40)$ $G(+40/-6)$ < 2	475 340 205 196 470 191 500	38, 33, 39 53 54 53 40, 33, 36, 39 53 41
ρ	0				0	751 ± 6	5.4	110 ± 10	1.3	πN	1029	$\pi^+\pi^+$ $\pi^+\pi^0$ $\pi^0\pi^0$ $\pi^-\pi^0$ $\pi^-\pi^+$ $\pi^-\pi^-$ neutrals		440	40
ρ_2	0				0	780	5.6	60	2.3	πN	1085	$\pi^+\pi^+$ $\pi^+\pi^0$ $\pi^0\pi^0$ neutrals		500	41
ρ_1	0				0	720	5.2	20	7	πN	975	$\pi^+\pi^+$ $\pi^+\pi^0$ $\pi^0\pi^0$ neutrals		440	40
ρ	+				0					π^+p	1066	$\pi^+\pi^+$ $\pi^+\pi^0$ $\pi^0\pi^0$ neutrals		495	39
ψ_4	-	2			0	760	5.4			πp	1310	$\pi^+\pi^-$	100	481	52
ψ_4	0				0	1055						$\pi^+\pi^+$	100	481	52
ψ_4	++				0	1590						$\pi^+\pi^+$	100	481	52
K_1^{*0}	0	$\frac{1}{2}$	> 1		1	730 ± 10	5.2	≤ 20	> 7	πp	1485	$K^+\pi^0$ $K^0\pi^0$ $(K\pi)^+$		96 97 92-101	50, 56 50, 56
K_1^{*+}	+				1										
δ	-	1 or 2			0	645 ± 25	4.5			πp	810	$\pi^+\pi^0$ $\pi^+\pi^+$ $\pi^0\pi^0$		350 345 350	43 43 43
δ	0				0										
δ	+				0										
α	0	1 or 2			0	625	4.5	< 80	> 1.7	pp		$\pi^+\pi^-\pi^0$ $\pi^+\pi^-\pi^+$ $\pi^+\pi^-\pi^-$		225 220	42 42
α	+				0										
ψ_2	-	2	0 or 2		0	605 ± 25	4.3	75	1.9	πp	1025	$\pi^+\pi^-$	100	326	45, 52
ψ_2	0				0	580	4.2				733	$\pi^+\pi^+$	100	301	52
ψ_2	++				0						1235	$\pi^+\pi^+$	100	326	52, 45
ζ	-	1			0	564 ± 9	4.0	< 43	> 3.2	πp	707	$\pi^+\pi^0$		289	44
ζ	0				0	541 ± 18	3.9			πp	672	$\pi^+\pi^0$ $\pi^+\pi^+$ $\pi^0\pi^0$		262 289	44 44
ζ	+				0					π^+p		$\pi^+\pi^0$ $\pi^+\pi^+$ $\pi^0\pi^0$		289	44
η	0	0	-+		0	548.5 ± 0.6	3.93	≤ 7	> 20	πp	685	$\pi^+\pi^+\pi^0$ $\pi^+\pi^-\pi^0$ $3\pi^0$ $\pi^0\pi^0\pi^0$ 2γ others	25 ± 10 7 ± 2 68 ± 10	135 270 144 414 549	46
η_2	0				0	520 ± 20	3.7	70 ± 30	2.0	πp	630	$\pi^+\pi^-$		240	47
ψ_2	-	2			0	440	3.1			πp	735	$\pi^+\pi^-$	100	160	52
ψ_2	0				0	420-440	3.1				515	$\pi^+\pi^+$	100	160	45, 52
ψ_2	++				0	440	3.1				975	$\pi^+\pi^+$	100	160	52
ϕ_1	0	0			0	395 ± 10	2.8	50 ± 20	2.8	πp	446	$\pi^+\pi^-$		115	47
ψ_1	-	2			0	330	2.4			πp	557	$\pi^+\pi^-$	100	50	52
ψ_1	0				0	330	2.4				346	$\pi^+\pi^+$	100	50	52
ψ_1	++				0	330	2.4				790	$\pi^+\pi^+$	100	50	52
ω_{ABC}	0	0			0	317 ± 6	2.3	≤ 16	> 9	pd		$\pi^+\pi^-$		38	49

Twenty-six states are listed. Seven are "exotic." It is now known that nineteen out of these twenty-six resonances do not exist!

The ace model was described in an eighty page CERN report:

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING
II *)

G. Zweig **)
CERN—Geneva

ABSTRACT

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number $1/3$ and is fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. Quantitative speculations are presented concerning resonances that have not as yet been definitively classified into representations of SU_3 . A weak interaction theory based on right and left handed aces is used to predict rates for $|\Delta S| = 1$ baryon leptonic decays. An experimental search for the aces is suggested.

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

**) This work was supported by the U.S. Air Force Office of Scientific Research and the National Academy of Sciences - National Research Council.

Quoting from the CERN report,

XII. CONCLUSIONS

The scheme we have outlined has given, in addition to what we already know from the Eightfold Way, a rather loose but unified structure to the mesons and baryons. In view of the extremely crude manner in which we have approached the problem, the results we have obtained seem somewhat miraculous.

A universality principle for the breaking of unitary symmetry by the strong interactions has been suggested. From this followed a qualitative understanding of the meson mass splittings in terms of the baryon mass spectrum, e.g., $m(\Lambda) > m(N)$ implies that $m(\varphi) > m(K^*) > m(\omega) \approx m(\rho)$. The proportionately larger mass splittings within the pseudoscalar meson octet have been explained. Mass formulae relating members of different representations have been suggested, e.g.,

$$(m^2(\omega) - m^2(\rho))/2 \approx m^2(\varphi) + m^2(\rho) - 2m^2(K^*) \quad ,$$

$$m^2(K^*) - m^2(\rho) \approx m^2(K) - m^2(\pi), \quad m(\Xi^-) - m(\Sigma^-) \approx m(\Xi_{\frac{1}{2}}^-) - m(\Sigma_{\frac{1}{2}}^-), \quad \text{etc.}$$

A universality principle for the breaking of unitary symmetry by the electromagnetic interactions has also been assumed. This has led to the qualitatively correct result that within any baryon charge multiplet, the more negative the particle, the heavier the mass. Electromagnetic mass splitting formulae relating members of different representations have been suggested, e.g.,

$$m(\Sigma^+) - m(\Sigma^-) \approx m(\Sigma_{\frac{1}{2}}^+) - m(\Sigma_{\frac{1}{2}}^-), \quad m(\Xi^-) - m(\Xi^0) \approx m(\Xi_{\frac{1}{2}}^-) - m(\Xi_{\frac{1}{2}}^0), \quad \text{etc.}$$

Nature's seeming choice of 1, 8, and 10-dimensional representations for baryons along with 1 and 8-dimensional representations for the mesons has been accounted for without dynamical or "bootstrap" considerations. The amount of octet-singlet ($\omega - \varphi$) mixing has also been predicted with algebraic techniques.

A pictorial method for determining strong interaction coupling constants has been presented. A unique baryon-baryon-pseudoscalar meson coupling has been suggested (F+D). We have found that $\varphi \rightarrow \rho \pi$ is forbidden to the order in which $m^2(\omega) = m^2(\rho)$. The interaction responsible for the splitting of the ω ρ masses has induced the decay $\varphi \rightarrow \rho \pi$ with a strength proportional to

$$\left[\frac{m^2(\omega) - m^2(\rho)}{m^2(\varphi) - m^2(\omega)} \right]^2 .$$

The quantum numbers available to a meson have been restricted to those which may be formed from the p, n, Λ and their antiparticles. The odd intrinsic parity of the pion and opposite nucleon parity fit naturally into the model.

The theory has been quantitatively applied to resonances that have not as yet been definitively classified into representations of SU_3 . $\Lambda_\gamma(1635)$, $K_\gamma(1318)$, $\eta_\alpha(775)$ are particles to be watched for.

Finally, a theory of the weak interactions has been considered. We assume that the weak decays of strongly interacting particles are induced by the weak decays of the aces which comprise them. From this followed :

- i) the conserved vector current theory:
 - ii) $|\Delta I| = 1/2$, $\Delta S/\Delta Q = +1$ for $|\Delta S| = 1$ leptonic decays:
 - iii) $\Sigma^- \rightarrow \Xi^0 + e^- + \nu$ is forbidden but $\Sigma^- \rightarrow \Xi^- + e^- + \nu$ is allowed.

Numerical results for hyperon β -decay have been presented.

There are, however, many unanswered questions. Are aces particles? If so, what are their interactions? Do aces bind to form only deuces and treys? What is the particle (or particles) that is responsible for binding the aces? Why must one work with masses for the baryons and mass squares for the mesons? And more generally, why does so simple a model yield such a good approximation to nature?

Additional results were presented in a series of lectures at the 1964 "Erice Summer School"¹⁰⁾. There, SU(6) symmetry was proposed for hadron classification.

The intellectual history of the quark model contains an enormous number of theoretical ideas and experimental results. There are many contradictions. If asked, "In this rich environment of fact and fiction, how could you find the quark?", I would reply "By having a basic commitment to reality." Each theoretical idea was tested by experiment, and experiments tested each other. For example, Matt Roos's 1963 Reviews of Modern Physics compilation of particles and their properties referred to several hundred experimental papers. I read essentially all of them, taking care to understand how each measurement was made. Then an accurate appraisal of the results of each experiment was possible. Rational choices between conflicting experiments usually could be made. Training in experimental physics was helpful in this process.

Many theories had to be judged in the face of insufficient experimental information. Theories which lacked predictive power, like Heisenberg's nonlinear spinor theory of matter, were discarded, not because they were necessarily incorrect, but because they were operationally useless. Theories of uncertain truth were compared with theories known to be either correct or incorrect. In this way it was possible to say that some theories "just didn't look right." Here training in theoretical physics was important.

In addition to this combination of experimental and theoretical skills, I possessed a rather unlikely combination of personality traits that was essential to my process of discovery. Near obsession with detail, with correctness and success coexisted with a much freer, imaginative and romantic nature.

Epilogue

The reaction of the theoretical physics community to the ace model was generally not benign. Getting the CERN report published in the form that I wanted was so difficult that I finally gave up trying^{13,14)}. When the physics department of a leading University was considering an appointment for me, their senior theorist, one of the most respected spokesmen for all of theoretical physics, blocked the appointment at a faculty meeting by passionately arguing that the ace model was the work of a "charlatan." The idea that hadrons, citizens of a nuclear democracy, were made of elementary particles with fractional quantum numbers did seem a bit rich. This idea, however, is apparently correct.

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