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14) See reference 5 for the proof of these theorems. Results of this nature were first obtained in reference 8 starting from Meiman's theorems (N. N. Meiman, Zh. Eksperim. i Teor. Fiz. 43 (1962) 2277 [English transl.: Soviet Phys.-JETP 16 (1963) 1609]). Similar result was also obtained by Y. S. Jin and S. W. MacDowell (reference 9) making use of the phase representation of the forward scattering amplitude.

A SYSTEMATICS OF HADRONS IN SUBNUCLEAR PHYSICS

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1.

With the recognition that the SU(3) symmetry is the dominant feature of the strong interactions, the main concern of the elementary particle theory has naturally become directed at the understanding of the internal symmetry of particles at a deeper level. An immediate question that arises in this regard is whether there are fundamental objects (such as triplets or quartets) of which all the known baryons and mesons are composed. These fundamental objects would be to the baryons and mesons what the nucleons are to the nuclei, and the electrons and nuclei are to the atoms. If that was really the case, it would certainly precipitate a new revolution in our conceptual image of the world. At the moment we can only hope that the question will be answered within the next ten to twenty years when the 100 GeV to 1000 GeV range accelerators will have been realized.

Even now, the amusing and rather embarassing success of the SU(6) theory [1] lends support to the existence of those fundamental objects. It is embarassing because this is basically a non-relativistic and static theory, and we do not know exactly how this can cover the realm of high energy relativistic phenomena.

Putting aside those theoretical difficulties mainly associated with relativity, let us make the working hypothesis that there are fundamental objects which are heavy ($\gg 1$ GeV), though not necessarily stable, and that inside each baryon or meson they are combined with a large binding energy, yet moving with non-relativistic velocities. Though this might look like a contradiction, at least it does not violate the uncertainty principle in non-relativistic quantum mechanics since the range of the binding forces ($10^{-14}-10^{-13}$ cm) is large compared

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to the Compton wave lengths of those constituents, and the strength of the forces can be arbitrarily adjusted. In other words, we have a model very similar to the atomic nuclei except for large binding energies. Theoretical justification of such a hypothesis must await future investigation.

In a previous article [2], we have put forward such a model with the following characteristic features.

1) There exist two fundamental fermion triplets t_1 and t_2 with charge assignments (1, 0, 0) and (0, −1, −1) for their three members. The baryons have the structure $\sim t_1 t_1 t_2$, and the mesons $\sim a t_1 \bar{t}_1 + b t_2 \bar{t}_2$.

2) To t_1 and t_2 are assigned "charm" charge $C = +1$ and -2 respectively. Thus the baryons and mesons (zero triality states) have $C = 0$. The primary binding forces acting on them are proportional to C. Let us imagine these forces to be mediated by a field (C-field). The resulting Coulomb-like energy though probably of finite range, then stabilizes the $C = 0$ ("uncharmed") systems against the $C \neq 0$ ("charmed") states, such as the triplets themselves.

u d s
1,0,0
1,0,0
0,-1,-1

3) The SU(6) symmetry can be brought in, with the Pauli principle taken into account, since the constituent particles are non-relativistic. In another paper, we also considered a three-triplet model, in which t_1, t_2 and t_3 have charge assignments (1, 0, 0), (1, 0, 0) and (0, −1, −1) respectively. This has the advantage that the baryon states (the 56-dimensional representation of SU(6)) may be realized with s-state triplets as $\sim t_1 t_2 t_3$.

The reasoning that has gone into the above stability problem is similar to the one used in nuclear physics in deriving the semi-empirical formula of Weizsäcker. The purpose of the present paper is to put this idea into a more precise form, even though the outcome should still be called at best semi-quantitative.

2.

Let us first consider states composed of an arbitrary number of t_1 and t_2, but without antiparticles $\bar{t}_1$ and $\bar{t}_2$. Their masses are M_1 and M_2, respectively, and the "charm" numbers 1 and −2, as was mentioned already. The pairwise interaction energy through the C-field will depend on the spatial configurations of the particles, but we will represent it, in the first approximation, by a constant V_c, as long as the size of the system is comparable with the range of the force. If the number of t_1's and t_2's are n_1 and n_2, respectively, the total energy of the system is

$$
\begin{aligned}
E(n_1, n_2) &= M_1 n_1 + M_2 n_2 + \\
&\quad + V_c \tfrac{1}{2} n_1 (n_1 - 1) + 4 V_c \tfrac{1}{2} n_2 (n_2 - 1) - 2 V_c n_1 n_2 \\
&= M_1 n_1 + M_2 n_2 + \tfrac{1}{2} V_c (n_1 - 2n_2)^2 - \tfrac{1}{2} V_c (n_1 + 4n_2) \qquad (1) \\
&= (M_1 - \tfrac{1}{2} V_c) n_1 + (M_2 - 2V_c) n_2 + \tfrac{1}{2} V_c C^2, \\
C &= n_1 - 2n_2.
\end{aligned}
$$

As expected, the leading quadratic term depends only on the total charm C. If V_c is sufficiently large, this will favor $C = 0$ as the lowest states, which means $n_1 = 2n_2$. Restricting ourselves to $C = 0$ states now, the remaining terms are linear in n_1 and n_2, implying a saturation property. With $n_1 = 2n_2$, we have

$$E(2n_2, n_2) = (2M_1 + M_2 - 3V_c) n_2. \qquad (2)$$

From the physical requirement that this increases with n_2 and that the baryon ($n_2 = 1$) be lighter than the triplets, we further need

$$M_1, M_2 > 2M_1 + M_2 - 3V_c > 0. \qquad (3)$$

Thus the energy surface in the $n_1 - n_2$ plane has a valley running along the line $C = n_1 - 2n_2 = 0$, and its level rises linearly with increasing coordinates. However, it will be further necessary to make sure that the $C = 0$ states are actually lower than their neighbors even for small n's. Namely

$$
\begin{aligned}
E(2n_2 \pm 1, n_2) &> E(2n_2, n_2), \\
E(2n_2, n_2 \pm 1) &> E(2n_2, n_2).
\end{aligned} \qquad (4)
$$

This gives two more conditions

$$V_c - M_1 > 0, \qquad 4V_c - M_2 > 0. \qquad (5)$$

Combining Eqs. (3) and (5), we obtain

$$3V_c - 2M_1 > M_2 - M_1 > 3(V_c - M_1) > 0. \qquad (6)$$

The second triplet, therefore, must be heavier than the first, but not too much heavier. This is because we have to maintain a balance between the energy due to rest masses and that due to interaction.

Eq. (1) may be expressed in terms of C and the baryon number B if we make an appropriate assignment: $B = x$ for t_1 and $B = y$ for t_2. Since the baryon $\sim t_1 t_1 t_2$ has $B = 1$, we require $2x+y = 1$. Possible choices given in ref. [2] are

$$(x, y) = (\tfrac{1}{3}, \tfrac{1}{3}) \quad \text{or} \quad (0, 1) \quad \text{or} \quad (1, -1). \tag{7}$$

The numbers n_1 and n_2 may be then expressed in terms of C and B as

$$\begin{aligned} n_1 &= 2B+yC \\ n_2 &= B-xC \end{aligned} \tag{8}$$

and thus

$$\begin{aligned} E(B, C) = \tfrac{1}{2}V_c C^2 &+ (2M_1+M_2-3V_c)B \\ &+ [(M_1-\tfrac{1}{2}V_c)-(2M_1+M_2-3V_c)x]C. \end{aligned} \tag{9}$$

At this point we should add a reservation that the linear terms in the above mass formula are not as meaningful as the leading quadratic terms since the effects depending on spatial configurations, such as those due to the finite range character of the C-field and the exchange energy, can be of the same order as the former.

3.

In order to consider the meson states, we will next bring in antiparticles as well in the picture. We make the basic assumption that a system consists of definite numbers of $n_1, \bar{n}_1, n_2, \bar{n}_2$ of $t_1, \bar{t}_1, t_2$ and $\bar{t}_2$. This means that *we regard pair creation and annihilation as forbidden processes*, which is consistent with our basic non-relativistic approach.

The formula corresponding to Eq. (1) becomes

$$\begin{aligned} E(n_1, \bar{n}_1, n_2, \bar{n}_2) = (M_1-\tfrac{1}{2}V_c)(n_1+\bar{n}_1)+(M_2-2V_c)(n_2+\bar{n}_2)+ \\ +\tfrac{1}{2}V_c C^2, \end{aligned} \tag{10}$$

$$C = n_1-\bar{n}_1-2(n_2-\bar{n}_2).$$

The requirement that $E > 0$ demands

$$M_1-\tfrac{1}{2}V_c > 0, \qquad M_2-2V_c > 0 \tag{11}$$

in contrast to Eq. (5), which was derived for the special case $\bar{n}_1 = \bar{n}_2 = 0$. We find, together with Eqs. (3) and (5),

$$M_1 > V_c-M_1 > M_2-2V_c > 0 \tag{12}$$

which replaces Eq. (6).

We will now relate the constants M_1, M_2 and V_c to the baryon $(t_1 t_1 t_2)$ and meson $(t_1 \bar{t}_1$ and $t_2 \bar{t}_2)$ masses m, μ_1 and μ_2:

$$\begin{aligned} m &= 2M_1+M_2-3V_c, \\ \mu_1 &= 2M_1-V_c, \\ \mu_2 &= 2M_2-4V_c, \end{aligned} \tag{13}$$

from which we obtain an identity

$$2\mu_1+\mu_2 = 2m. \tag{14}$$

Because of this, we cannot determine the three unknowns M_1, M_2, V_c uniquely. Instead, we can express Eq. (10) in terms of μ_1 and μ_2:

$$E(n_1, \bar{n}_1, n_2, \bar{n}_2) = \tfrac{1}{2}\mu_1(n_1+\bar{n}_1)+\tfrac{1}{2}\mu_2(n_2+\bar{n}_2)+\tfrac{1}{2}V_c C^2. \tag{15}$$

Turning to the relation (14), we put $m \sim 1.2$ GeV, $\mu_1 \sim 600$ MeV $= \frac{1}{2}m$ corresponding to the average baryon and meson masses, and *predict* a value

$$\mu_2 \sim m \sim 2\mu_1 \tag{16}$$

for the second meson. This is not an unreasonable value in view of the fact that a large number of unidentified meson resonances seem to exist in this energy range. Eq. (15) reduces then to the simple form

$$E(n_1, \bar{n}_1, n_2, \bar{n}_2) = \tfrac{1}{2}\mu_1[n_1+\bar{n}_1+2(n_2+\bar{n}_2)]+\tfrac{1}{2}V_c^2. \tag{17}$$

It is rather surprising that such a naive picture as ours can yield non-trivial and qualitatively reasonable results.

By way of a remark, we note from Eq. (13) that

$$\begin{aligned} M_1 &= \tfrac{1}{2}V_c+\tfrac{1}{2}\mu_1 \sim \tfrac{1}{2}V_c, \\ M_2 &= 2V_c+\tfrac{1}{2}\mu_2 \sim 2V_c \sim 4M_1 \end{aligned} \tag{18}$$

since $V_c \gg \mu_1, \mu_2$ by assumption. Interestingly enough, the above relation admits the interpretation that the mass of each triplet is made up of a self-energy due to the C-field plus a small "bare mass" $\frac{1}{2}\mu$.

4.

We will now turn to the three-triplet model [3] proposed as an alternative to the two-triplet model. The three triplets t_1, t_2 and t_3 altogether contain nine fermions $T_{i\alpha}$, $i, \alpha = 1, 2, 3$, where the index i distinguishes different triplets, and α the different members of a triplet. Two different SU(3) operations, called SU(3)′ and SU(3)″, are introduced, acting respectively on α and i, and in these spaces $T_{i\alpha}$ behave as a representation (3, 3*). The electric charge is assigned to each particle according to

$$Q = I_3' + \tfrac{1}{2}Y' + I_3'' + \tfrac{1}{2}Y'' \tag{19}$$

which takes integral values. In fact both $T_{1\alpha}$ and $T_{2\alpha}$ have the assignment (1, 0, 0), and $T_{3\alpha}$ have (0, −1, −1), exactly like t_1 and t_2 of the previous two-triplet model.

An important difference from the two-triplet case is that instead of the charm gauge group $U(1)$, we have the group SU(3)″. The charm gauge field C must then be replaced by an octet of gauge fields G_μ, $\mu = 1, \ldots, 8$, coupled to the infinitesimal SU(3)″ generators (currents) λ''_μ of the triplets, with a strength g. For a system containing altogether N particles, the exchange of such fields between a pair then results in an interaction energy

$$\begin{aligned} V_G = +g^2 \sum_{n>m} \lambda''^{(n)}_\mu \cdot \lambda''^{(m)}_\mu &= \tfrac{1}{2}g^2\Big[\sum_{n=1}^{N} \lambda''^{(n)}_\mu\Big]\Big[\sum_{m=1}^{N} \lambda''^{(m)}_\mu\Big] - \tfrac{1}{2}g^2 \sum_{n=1}^{N} \lambda''^{(n)}_\mu \lambda''^{(n)}_\mu \\ &= \tfrac{1}{2}g^2[C_2 - NC_{20}], \end{aligned} \tag{20}$$

where $\lambda^{(n)}_\mu$ refers to the n-th particle, C_2 is the quadratic Casimir operator of SU(3), and $C_{20} = 4/3$ is its value for a triplet representation $D(1, 0)$ or $D(0, 1)$. In general C_2 is given by

$$C_2(l_1, l_2) = \tfrac{1}{3}(l_1^2 + l_1 l_2 + l_2^2) + (l_1 + l_2) \tag{21}$$

for a representation $D(l_1, l_2)$.

Note that the only dependence on the total number N of constituents appears in the second term of Eq. (17).

We add to V_G the rest masses (M = common mass), and obtain the total energy

$$E = (M - \tfrac{1}{2}C_{20}g^2)N + \tfrac{1}{2}g^2 C_2. \tag{22}$$

Bound states are characterized by $V_G < 0$, and the low lying states by the smallest value of C_2, namely $C_2 = 0$ for the singlet $D(0, 0)$. For the latter, E is simply proportional to the total number N of constituents, starting with the meson $(N = 2) \sim t_1\bar{t}_1 + t_2\bar{t}_2 + t_3\bar{t}_3$ and the baryon $(N = 3) \sim t_1 t_2 t_3$ (antisymmetric combination). Their masses are thus related by

$$\mu = 2(M - \tfrac{1}{2}C_{20}g^2) = \tfrac{2}{3}m, \tag{23}$$

and Eq. (22) becomes

$$E = \tfrac{1}{2}\mu N + \tfrac{1}{2}g^2 C_2. \tag{24}$$

These are to be compared with Eqs. (14) and (15). Because of the high symmetry among the three triplets, we have found only one set of mesons with $N = 2$. In any case, the energy is simply proportional to the total number of constituents as long as $C_2 = 0$, as if it were made up of non-interacting basic units of mass $\frac{1}{2}\mu$.

5.

Having disposed of the gross mass spectrum of many-triplet compound systems, we now turn our attention to the "fine structure" of low lying states, which in our view comprise all the mesons and baryon resonances known so far. In all probability, however, our crude qualitative arguments are not really satisfactory for discussing these details. We will therefore restrict ourselves to general remarks only.

Because of our basic assumptions about the superstrong interactions and the static behaviour of particles, the dynamics we have been dealing with so far does not depend on the spin and the SU(3) spin variables, therefore the system possesses the symmetry of superstrong interactions, the SU(6) symmetry of combined spin and SU(3) spin, and the symmetry of orbital angular momentum. The overall Pauli principle imposes constraints among these symmetries, and thereby single out certain SU(6) and orbital states for the lowest configuration with respect to the superstrong interaction. The general classification of these states can be done as in the case of nuclear and atomic physics, but this will be beyond the scope of the present paper.

In the three-triplet model, however, the problem is relatively simple if we take only s-state triplets. The low lying three particle configuration is a SU(3)'' singlet, so the baryon must go into a complete symmetric SU(6) representation 56. No other states are possible without changing the spatial configuration, but this will cause some change in the superstrong interaction. For the mesons, we obviously obtain 36 = 35+1 SU(6) states which are degenerate. These results are in accordance with those of the original SU(6) theory, as well as its "relativistic" version.

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$36 = 3.\bar{9} + 1.9$

We must next discuss the two additional effects which do exist and tend to upset the symmetries. One arises from the internal motion of particles, and the other from the presence of virtual mesons. Contrary to the prevalent view, we regard the mesons as perturbing forces rather than the decisive factors in the physics of hadrons. Since the strong interactions are then merely first forbidden processes, so to speak, the meson and baryon resonances are really bound states decaying via violation of superstrong interaction symmetry. Nevertheless, these secondary effects can affect, and may even decide, the "fine structure" of low lying states. Perhaps we may compare the situation to the electronic levels of an atom where the main spectrum is determined by the static Coulomb force, and both the fine structure and the photon emission processes are higher order effects. In this sense, we do not necessarily find a contradiction between the present approach and the conventional strong interaction theory as far as the low lying states are concerned.

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The reason we consider the strong interaction as generally symmetry breaking is that the virtual exchange of 36 virtual mesons do not possess an SU(6) symmetric form. An ideal SU(6) symmetric interaction would involve the 35 generators χ_μ as in Eq. (20):

$$V \sim \pm g^2 \sum_{n>m} \chi_\mu^{(n)} \chi_\mu^{(m)} = \pm \frac{g^2}{8} \sum_{n>m} \left[\tfrac{2}{3}\sigma_i^{(n)}\sigma_i^{(m)} + \lambda_\alpha^{(n)}\lambda_\alpha^{(m)} + \sigma_i^{(n)}\sigma_i^{(m)}\lambda_\alpha^{(n)}\lambda_\alpha^{(m)}\right]. \tag{25}$$

Viewed as a static force, this requires an exchange of 35 scalar and axial vector mesons (opposite parity to the known meson multiplet!)

if the relative signs of the various terms are to be correctly maintained for both particle-particle and particle-antiparticle interactions. [For processes involving meson-baryon scattering, however, Capps [4] and Belinfante and Cutkosky [5] have shown their compatibility with SU(6).]

Next consider the effect of the internal motion. This disturbs the basic symmetry in two senses. It mixes the Dirac spinor components, introducing corrections to the static superstrong forces. Further it simply adds the kinetic energy of orbital motion to the system. As far as the symmetry is concerned, these perturbations act like adding a neutral singlet meson with a suitable spin-parity. Its order of magnitude will depend on the internal velocity v of the particles, which should be of the order $1/MR$ where R is the size of the system. If we take this correction to be of the order $Mv^2 \sim 100$ MeV, and $R \sim 1/M$, we obtain the estimation

$$M \sim 10m, \qquad \frac{v}{c} \sim \frac{1}{10}$$

as we did before [2].

6.

Finally we would like to comment on some obvious difficulties and intriguing problems concerning our model of the subnuclear structure of hadrons.

a) What is the origin of superstrong interactions? Are these another kind of vector fields or something entirely new? If they are ordinary fields, their range must be at least of the order of the baryon size, and moreover sufficiently smooth and well-behaved in order to keep the kinetic energy small. It is conceivable that no single or a relatively few well defined meson states are responsible for this. A direct confirmation of such interactions would be difficult.

b) The magnetic moments of baryons, for example, agree closely with the SU(6) symmetry, yet obviously the bulk of contributions come from the meson cloud. This means that regardless of whether the meson cloud obeys SU(6) symmetry or not, the baryon should not be considered as composed of three bare triplets without structure. How, then, can we justify our picture that each system, including the meson, is composed

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of a definite number of triplets? The answer to this probably should be that the quantities like charm are at any instant well localized at a definite number of centers in space, and these centers are accompanied by large concentrations of energy, moving with slow velocities; whereas the quantities like ordinary charge are more uniformly spread out and carried by faster moving matter. In order to test such a picture experimentally, we would have to use some phenomena which depend on the energy distribution, the correlation functions of charges and energies at different points, the internal velocity of particles, etc.

c) The notion that decays and resonances are actually forbidden processes was first recognized as a surprising paradox in the process of adapting SU(6) to relativity. In our view, this is not only natural, but also simplifies the whole picture. We should be able to discuss the classes of first forbidden, second forbidden, etc. transitions, and they will be accessible to experimental test [6]. For this we should look especially for small, inconspicuous bumps in cross sections, many particle decay modes, and relatively rare events.

d) It has been widely speculated that an axial vector current conservation as relativistic chival symmetry has physical significance. If this is actually the case, it is probably beyond the capacity of our extreme static approach, since we have first to explain away the large masses of triplets, even though we can formally apply group theoretical arguments and the Goldberger-Treiman type relations to individual problems.

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Zitieden: HH

A LORENTZ COVARIANT SUPERMULTIPLET SCHEME FOR STRONG INTERACTIONS

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1. INTRODUCTION

We would like to describe in this article a Lorentz-covariant scheme for elementary particle interactions, which reproduces the successful features of non-relativistic SU(6)-models [1], and gives further interesting results. We know that it is apparently not possible to have a reasonable theory with finite supermultiplets which is Lorentz-invariant, and which also complies with all the basic assumptions of field theory or of dispersion theory [2]. Therefore, we aim only at an approximate scheme for S-matrix elements and form factors.

As a starting point, we assume that the fields describing the asymptotic, noninteracting particles are tensors of $U_{\mathscr{L}}(12)$[U(6, 6)] [3–5]. However, we require these tensors to satisfy the Bargmann-Wigner equations [6], which are not covariant with respect to $U_{\mathscr{L}}(12)$, but which define the physical particles in just such a way that we have the supermultiplet structure of U(6). Several authors [5] have used the modified $U_{\mathscr{L}}(12)$-tensors in order to construct vertex-parts and four-particle amplitudes which are formally invariant under $U_{\mathscr{L}}(12)$, except for the intrinsic symmetry breaking due to the equations of motion. This scheme turns out to be too restrictive [7, 8].

We have proposed, therefore, a more general scheme [9, 10] where the $U_{\mathscr{L}}(12)$-invariance of a given amplitude is broken, not only by the Bargmann-Wigner equations, but also by the insertion of momentum spurions

$$S = (\Gamma_\mu \gamma_\mu) \otimes 1 \tag{1}$$

in arbitrary order. In Eq. (1), the vector Γ_μ should be constructed out