

## FROM WIGNER'S SUPERMULTIPLY THEORY TO QUANTUM CHROMODYNAMICS

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### Abstract

The breadth of Eugene Wigner's interests and contributions is amazing and humbling. At different times in his life he did seminal work in areas as diverse as pure mathematics and chemical engineering. His seminal research in physics is, of course, the best known. In this talk I first describe Wigner's supermultiplet theory of 1936 using the approximate symmetry of the nuclear Hamiltonian under a combined spin-isospin symmetry to describe the spectroscopy of stable nuclei up to about the nucleus molybdenum. I then show how Wigner's ideas of 1936 have had far reaching and unexpected implications: his ideas led to the discovery of the color degree of freedom for quarks and to the symmetric quark model of baryons which is the basis of baryon spectroscopy. I conclude by pointing out that the color degree of freedom, made into a local symmetry using Yang-Mills theory, leads to the gauge theory of color, quantum chromodynamics, which is our present theory of the strong interactions.

I am very happy to participate in this Centennial Conference to remember and honor the life and work of Eugene Wigner. I start by giving some reminiscences of my contacts with Wigner. When I was a first year graduate student I approached

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Wigner at the daily afternoon tea in Fine Hall to ask a question. I don't remember the question, but I do remember the kindness with which Wigner treated me. He asked me to join him in his office, which was also in Fine Hall, and gave me a careful and full answer to my question. Once on a winter day I saw Wigner, who was slight in physique, almost blown in by the wind as he entered the side door of the passage that connected Palmer Lab with Fine Hall. I had the pleasure to take two courses with Wigner during my studies, Kinetic Theory and Group Theory.

Wigner was unfailingly polite. When Wigner had to leave a seminar at the Institute for Advanced Study before it had ended, he would carefully and quietly gather his belongings and try to tiptoe out of the room. Generally this took some time during which all eyes were on him. His attempt to be unobtrusive had the opposite effect. On one occasion, Wigner asked several questions of one of his nuclear physics graduate students during his seminar. His student was impatient and answered grudgingly. Wigner shrugged, stopped asking questions and said "I don't want to slow down the progress of science."

Among the historic events with which Wigner was involved, two have not been mentioned here, so I will recall them now. One is that Wigner joined Leo Szilard in driving to Long Island, where Einstein was sailing, to ask him to write to President Franklin Roosevelt to make him aware of the potential of a nuclear weapon. The other is that Wigner had the foresight to bring a bottle of chianti to the squash court under Stagg Field at the University of Chicago to celebrate the nuclear reactor built under the direction of Enrico Fermi becoming critical for the first time. All present signed the wicker basket holding the bottle. I don't know where this historic basket is now.

In the summer of 1962 there was a NATO summer school organized by Feza Gürsey at what then was called the Robert College in Bebek, Turkey outside of Istanbul. The proceedings are still worth reading.[1] Wigner gave a series of lectures on the representations of the inhomogeneous Lorentz group of relevance to elementary particles, with particular emphasis on the extension of the group to take account of the discrete elements, parity, time reversal, and spacetime reversal.[2] Of course Wigner himself had introduced these discrete symmetries into quantum mechanics. In these lectures, Wigner found some unusual representations in which the dimension

of the representations of the discrete elements is doubled. As the senior American physicist, Wigner felt responsible to provide a report on the summer school for publication in *Physics Today*. He asked me to write a first draft of the report, which duly appeared.[3] After leaving Bebek we both traveled to Trieste, where lectures were being given at what became the International Center for Theoretical Physics founded by Abdus Salam. Among the lecturers was Julian Schwinger, who gave his solution of two-dimensional quantum electrodynamics. Both Wigner and I had to leave early one morning to fly to Venice. We were out on the road by the hotel, cold and hungry, waiting for a late cab to arrive. At the airport there was no time to eat. We were flying in an old DC3, which lands on its tail and wing wheels. I found some old sticks of chewing gum and shared them with Wigner, so we could get some calories into our bodies.

Now I turn to the physics part of my talk. Wigner was the first to combine spacetime and internal symmetries into a larger symmetry group whose approximate validity leads to new predictions about measurable quantities. In Wigner's paper of 1936[4] he discusses the various types of nuclear forces, space dependent forces, space and spin dependent forces, space and isospin dependent forces, and finally, space, spin and isospin dependent forces. Bear in mind that Werner Heisenberg had introduced the concept of isospin, that the proton and neutron can be considered as members of an SU(2) doublet, the nucleon, just four years before, in 1932. The spin states of a spin 1/2 particle can also be considered as an SU(2) doublet. Wigner's seminal idea was to combine the two groups into their associative covering group, SU(4), and to explore the consequences of an approximate SU(4) spin-isospin symmetry. Thus Wigner gave the first example of a symmetry that combined spacetime and internal symmetries. What Wigner did was to combine the spin-1/2 doublet

$$\left(\frac{1}{2}\right) \sim \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \sim 2_S \text{ in } \text{SU}(2)_S \quad (1)$$

with the isospin-1/2 doublet

$$(N) \sim \begin{pmatrix} p \\ n \end{pmatrix} \sim 2_I \text{ in } \text{SU}(2)_I \quad (2)$$

to get a quartet in the larger group

$$\begin{pmatrix} p \uparrow \\ p \downarrow \\ n \uparrow \\ n \downarrow \end{pmatrix} \sim 4_{SI} \text{ in } SU(4)_{SI}. \quad (3)$$

One can reverse the process and decompose the quartet into a product of doublets

$$4_{SI} \rightarrow 2_S \otimes 2_I \text{ under } SU(4)_{SI} \rightarrow SU(2)_S \otimes SU(2)_I. \quad (4)$$

Wigner showed that his larger symmetry gave a good fit to nuclear states up to molybdenum.

Here we have Wigner as the master of the use of group theory in physics. On the one hand, he used powerful theorems due to G. Frobenius (whom I have heard him call “old man Frobenius”), E. Cartan, I. Schur and H. Weyl. On the other hand, Wigner as a profound physicist used the group theory results to confront experimental data.

The next step in the story of the fruits of Wigner’s idea to combine spin and isospin symmetry was taken by Feza Gürsey and Luigi A. Radicati[5] in 1964, soon after the quark model was introduced independently by Murray Gell-Mann[6] and by George Zweig[7]. Gürsey and Radicati transplanted Wigner’s idea to elementary particle physics using the  $SU(2)_S$  symmetry and what now we call the flavor symmetry  $SU(3)_F$  and combining them to the associative covering group  $SU(6)_{SF}$ . Their construction was closely analogous to Wigner’s work of 1936. We have

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \sim \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \sim 2_S \text{ in } SU(2)_S \quad (5)$$

combined with the flavor-triplet

$$(q) \sim \begin{pmatrix} u \\ d \\ s \end{pmatrix} \sim 3_F \text{ in } SU(3)_F \quad (6)$$

to get a sextet in the larger group

$$\left(q\frac{1}{2}\right) \sim \begin{pmatrix} u \uparrow \\ u \downarrow \\ d \uparrow \\ d \downarrow \\ s \uparrow \\ s \downarrow \end{pmatrix} \sim 6_{SF} \text{ in } SU(6)_{SF}. \quad (7)$$

Again the larger representation decomposes to a product of representations of each of the constituent groups,

$$6_{SF} \rightarrow 2_S \otimes 3_F \text{ under } SU(6)_{SF} \rightarrow SU(2)_S \otimes SU(3)_F. \quad (8)$$

Great skepticism about quarks reigned in the physics community. Fractionally particles had never been seen; indeed they have not been seen to this day. Gell-Mann himself was ambiguous about the reality of quarks.

The  $SU(6)_{SF}$  theory had some striking successes: the lowest-mass mesons, the pseudoscalar octet and the vector nonet, fit precisely into a  $35 + 1$  of  $SU(6)_{SF}$ . The lowest-mass baryons fit exactly into a  $56$  of  $SU(6)_{SF}$ . Gürsey and Radicati derived mass formulas for each supermultiplet and these mass formulas agreed well with data. For the mesons,

$$q\bar{q} \sim 6 \otimes 6^* = 1 + 35 \text{ in } SU(6)_{SF} \quad (9)$$

$$1 \rightarrow (1, 0); \quad 35 \rightarrow (8, 0) + (1 + 8, 1) \text{ under } SU(3)_{FS} \otimes SU(2)_S. \quad (10)$$

There is nothing to raise concern about the placement of the mesons in the  $SU(6)$  theory.

The baryons are a different story. Since three quarks are bound in a baryon (in what we now call the “constituent” quark model), the possibilities are

$$qqq \sim 6 \otimes 6 \otimes 6 = 56 + 70 + 70 + 20. \quad (11)$$

The  $56$  is totally symmetric under permutations of the quarks; the  $70$ ’s have mixed symmetry and the  $20$  is totally antisymmetric.

The spin-statistics theorem[8] states that integer-spin particles form symmetric states under permutations and odd-half-integer-spin particles form antisymmetric states. Since the quarks are assigned spin-1/2 in order to give the odd-half-integer spin of the baryons, the spin-statistics theorem requires the quarks to be in the 20. Bunji Sakita[9] made this choice. As we will see, both the observed spectrum of lowlying baryons and the ratio of the magnetic moments of the proton and neutron rule out this assignment. Gürsey and Radicati chose the 56, which fits the baryons beautifully. This choice, symmetric under permutations, violates the spin-statistics theorem and presents a paradox: quarks as spin-1/2 particles should be fermions and should obey the Pauli exclusion principle, yet the quarks are symmetric under permutations in the 56. This paradox compounded the problems of the quark model and greatly increased the reluctance of the physics community to accept the quark model. Unobserved fractionally charged particles were bad enough. Particles that violated the exclusion principle were just too incredible.

Nonetheless, in addition to the success of the  $SU(6)$  theory for the lowlying baryons, there was another striking result that agreed with experiment. M.A.B. Baqi Bég, Benjamin W. Lee and Abraham Pais[10] and, independently, Sakita[11] calculated the ratio of the magnetic moments of the proton and neutron and found the simple result

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \quad (12)$$

using pure  $SU(6)$  group theory. This result agrees with experiment to within 3%! All previous calculations using meson cloud effects had failed utterly. Nobody had realized that the ratio had such a simple value. The magnetic moment calculation gave heart to those who believed that the  $SU(6)$  theory was on the right track.

The first solution to the spin-statistics paradox was provided by parastatistics. Parastatistics had been introduced by H.S. Green[12] in 1953. Green noticed that the commutation relations between the number operator and the annihilation and creation operators are the same for both bosons and fermions,

$$[n_k, a_l^\dagger]_- = \delta_{kl} a_l^\dagger, \quad [n_k, a_l]_- = -\delta_{kl} a_l \quad (13)$$

The number operator can be written

$$n_k = (1/2)[a_k^\dagger, a_k]_\pm + \text{const}, \quad (14)$$

where the anticommutator (commutator) is for the Bose (Fermi) case. He realized that he could generalize each of these types of particle statistics to a family, labeled by the order  $p$ , where  $p = 1, 2, 3, \dots$  and, in each case,  $p = 1$  is the usual statistics. If these expressions are inserted in the number operator-creation operator commutation relation, the resulting relation is *trilinear* in the annihilation and creation operators. Polarizing the number operator to get the transition operator  $n_{kl}$  which annihilates a free particle in state  $k$  and creates one in state  $l$  leads to Green's trilinear commutation relation for his parabose and parafermi statistics,

$$[[a_k^\dagger, a_l]_{\pm}, a_m^\dagger]_{-} = 2\delta_{lm}a_k^\dagger \quad (15)$$

Since these rules are trilinear, the usual vacuum condition,

$$a_k|0\rangle = 0, \quad (16)$$

does not suffice to allow calculation of matrix elements of the  $a$ 's and  $a^\dagger$ 's; a condition on one-particle states must be added,

$$a_k a_l^\dagger |0\rangle = \delta_{kl} |0\rangle. \quad (17)$$

Green found an infinite set of solutions of his commutation rules, one for each integer, by giving an ansatz which he expressed in terms of Bose and Fermi operators.

Let

$$a_k^\dagger = \sum_{p=1}^n b_k^{(\alpha)\dagger}, \quad a_k = \sum_{p=1}^n b_k^{(\alpha)}, \quad (18)$$

and let the  $b_k^{(\alpha)}$  and  $b_k^{(\beta)\dagger}$  be Bose (Fermi) operators for  $\alpha = \beta$  but anticommute (commute) for  $\alpha \neq \beta$  for the “parabose” (“parafermi”) cases. This ansatz clearly satisfies Green's relation. The integer  $p$  is the order of the parastatistics. The physical interpretation of  $p$  is that, for parabosons,  $p$  is the maximum number of particles that can occupy an antisymmetric state; for parafermions,  $p$  is the maximum number of particles that can occupy a symmetric state (in particular, the maximum number which can occupy the same state). The case  $p = 1$  corresponds to the usual Bose or Fermi statistics. Later, Messiah and I[13] proved that Green's ansatz gives all Fock-like solutions of Green's commutation rules. Local observables have a form analogous to the usual ones; for example, the local current for a spin-1/2 theory

is  $j_\mu = (1/2)[\bar{\psi}(x), \psi(x)]_-$ . From Green's ansatz, it is clear that the squares of all norms of states are positive, since sums of Bose or Fermi operators give positive norms. Thus parastatistics gives a set of orthodox theories. Parastatistics is one of the possibilities found by Doplicher, Haag and Roberts[14] in a general study of particle statistics using algebraic field theory methods. A good review of this work is in Haag's book[15].

Note that Wigner[16] anticipated parastatistics in his study of generalizations of the harmonic oscillator. His case corresponds to parabose statistics of order 2 for a single oscillator.

The spin-statistics theorem generalizes in the context of parastatistics. The theorem becomes *Given the choice between parabose and parafermi statistics*, parabose particles must have integer spin and parafermi particles must have odd-half-integer spin[17].

My colleagues at Maryland had asked me to invite Gürsey, who in the summer of 1964, was in Brookhaven, to give a seminar at Maryland. When I called him, he said that he and Radicati had just finished some interesting work that would soon appear in Physical Review Letters. I was to be on leave at the Institute for Advanced Study. When I arrived there was great interest in the  $SU(6)$  theory. Ben Lee showed me the magnetic moment calculation which really convinced me that the  $SU(6)$  theory was correct. Because I had been working on parastatistics with A.M.L. Messiah at Saclay, I immediately realized that if the quarks obeyed parafermi statistics of order 3 the contradiction with the spin-statistics theory would be removed.

I was able to show that three parafermi quarks of order three can be in a state that is totally symmetric under permutations of the visible degrees of freedom, space, spin and unitary spin, and that the necessary antisymmetry is provided by the parastatistics. Further, I showed that the composite operator of three such quarks behaves as a fermion as the nucleon must. The first sentence of this paper starts with the words "Wigner's supermultiplet, ..." [18] I redid the magnetic moment calculation using a concrete composite model for the nucleon with creation operators, all in the  $S$ -state for the quarks, since their parafermi nature takes care of the antisymmetrization. The calculation is elementary. The proton with spin up must



have quark content  $uud$  and spin content  $\uparrow\uparrow\downarrow$ . Thus the possible creation operators that can enter are  $u_\uparrow$ ,  $u_\downarrow$ ,  $d_\uparrow$ , and  $d_\downarrow$ . For the proton with spin up, the only products of creation operators are  $u_\uparrow u_\uparrow d_\downarrow$  and  $u_\uparrow u_\downarrow d_\uparrow$ .

For the fermi case that Sakita chose, the creation operators anticommute,

$$[u, d]_+ = 0, \text{ etc.} \quad (19)$$

Then the only possibility is

$$|p_\uparrow\rangle = |u_\uparrow u_\downarrow d_\uparrow\rangle. \quad (20)$$

Then the contributions of the  $u$  quarks to the magnetic moment cancel and the result is

$$\mu_p = \mu_d. \quad (21)$$

Since the  $d$  has negative charge, the proton magnetic moment is antiparallel to its spin; it points the wrong way! The same is true for the magnetic moment of the neutron.

The bose case (always keeping in mind that the overall antisymmetry is provided by the parastatistics) is a bit more complicated. Make a neutral “core” with  $I = S = 0$ ,  $u_\uparrow d_\downarrow - u_\downarrow d_\uparrow$ . Then the proton with spin up is

$$|p_\uparrow\rangle = \frac{1}{\sqrt{3}} |u_\uparrow (u_\uparrow d_\downarrow - u_\downarrow d_\uparrow)\rangle \quad (22)$$

In terms of the spins carried by the  $u$  and  $d$  quarks, the magnetic moment operator  $\mu$  is

$$\mu = 2\mu_0 \left( \frac{2}{3} S_u - \frac{1}{3} S_d \right), \quad (23)$$

where  $\mu_0$  is the Bohr magneton of the quark, and  $S_u$  and  $S_d$  are the spin operators of the quarks. The coefficients of the spin operators are the electric charges of the quarks. Then

$$\mu_p = \langle p_\uparrow | \mu | p_\uparrow \rangle = 2\mu_0 \frac{1}{3} \left\{ 2 \left[ \frac{2}{3} \cdot 1 + \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{2}\right) \right] + \left(-\frac{1}{3}\right) \cdot \left(\frac{1}{2}\right) \right\} = \mu_0. \quad (24)$$

Here the  $1/3$  factor in front of the curly bracket is the square of the normalization factor of the proton, the factor 2 in front of the square bracket is the bose factor from two  $u$  quarks, the fractions of  $2/3$  and  $-1/3$  are the charges of the quarks,

and the factors of 1,  $-1/2$  and  $1/2$  are the  $z$ -components of the quark spins. The corresponding result for the neutron is

$$\mu_n = -\frac{2}{3}\mu_0. \quad (25)$$

Thus, as mentioned above,

$$\frac{\mu_p}{\mu_n} = -\frac{2}{3}. \quad (26)$$

I pointed out that if the quarks couple minimally, then the mass of the quark would be about  $1/3$  of the nucleon mass. This is the value that is now taken for the “constituent” mass of the  $u$  and  $d$  quarks.

Now, thirty eight years after the fact, it is difficult to recover the mind set of the fall of 1964 when the developments I am describing took place. At the time several solutions to the statistics paradox were suggested. These include

- Parastatistics, which is equivalent to color as a classification symmetry
- Three integer charged triplets
- Quarks are a mathematical fiction
- Quarks are indeed fermions with a complicated ground state wavefunction
- Extra quark-antiquark pairs
- More exotic possibilities

Parastatistics and its equivalence to color as a classification symmetry is what I am discussing here. Three integer charged triplets were first suggested by Yoichiro Nambu[19] and further developed in [20] and [21]. Also see [22]. I will discuss this case later. The possibility that quarks are just a mathematical fiction is due to Gell-Mann in his original paper. This would obviate the statistics paradox at the cost of making quarks non-physical. Both Nambu and I took an unequivocal stand that quarks are real and physical. Richard H. Dalitz, for some years the rapporteur on hadron spectroscopy at international conferences, was a leader of those who preferred to assume quarks are fermions (without an additional degree of freedom) and that the statistics paradox would be avoided by having an antisymmetric space ground state wavefunction. Because there are only two (nonrelativistic) space coordinates available when the center-of-mass coordinate is eliminated, the simplest

antisymmetric scalar wavefunction has degree 6 in the coordinates. This is implausible in view of theorems that require the ground state wavefunction to be nodeless. In addition, if the ground state wave function has this complicated form, then there is no clear candidate for the excited states. As we will see, the parastatistics theory, in which the ground state wavefunction is nodeless, leads to a simple model for the excited states. Finally, if the ground state wave function has nodes, then there should be zeroes in the electric and magnetic form factors of the nucleon. No such zeroes have ever been found. The possibility of extra quark-antiquark pairs is likely to lead to “exploding  $SU(3)$  representations,” which have not been seen. Of course we now believe that the nucleon and other hadronic states are not solely described by their constituent quark content; they also have terms with extra quark-antiquark pairs as well as gluons; however the classification of the hadronic states can still be made in terms of the quantum numbers of their constituent quarks.

The quark statistics is irrelevant for states of mesons. For baryons, statistics is crucial. I proposed that the baryons should be described using the “symmetric” quark model in which the states are totally symmetric in terms of the visible space, spin and unitary spin degrees of freedom[18]. The antisymmetry of the quarks is taken care of by the parastatistics of order 3. I developed an atomic model in which the higher states of baryons have the quarks excited into higher orbital states, starting with the  $p$  state of opposite parity. (Recall that parity was introduced by Wigner, as was time reversal.) I gave a table of excited states in my paper of 1964. Gabriel Karl and Obryk[23] later corrected some of the states.

Since I was at the Institute for Advanced Study, I was eager to hear the opinion of J. Robert Oppenheimer. I gave him a preprint of my paper before leaving for the Eastern Theoretical Physics Conference, which we both attended, that was to take place at my home university, the University of Maryland in College Park. When I saw Oppenheimer I asked if he had read my paper. He said

“Greenberg, your paper is beautiful,”

That made my spirits rise; however he continued

“but I don’t believe a word of it.”

Then my spirits came down again.

I was asked to speak about my paraquark model at Harvard. In the parking lot after the talk, Julian Schwinger made the prescient remark that the implicit degree of freedom implicit in the parastatistics model should play a dynamical role. To my regret I did not follow up on Schwinger's remark, which pointed toward quantum chromodynamics.

For some years there was resistance to quarks and color.

- Unobserved fractionally-charged quarks seemed outrageous.
- Gell-Mann's ambiguous position cast doubt on the reality of quarks.
- A new hidden degree of freedom seemed doubly outrageous.

The arguments for quarks and color included baryon and meson spectroscopy, the magnetic moments of the nucleon, the Zweig rule which predicted suppression of decays without connected quark line graphs, the parton model, the  $J/\Psi$  and its friends, and the  $\pi^0 \rightarrow \gamma\gamma$  decay and the axial anomaly.

Marvin Resnikoff and I gave a detailed fit of the baryons in the  $(\mathbf{56}, \mathbf{L} = 0)$  and the  $(\mathbf{70}, \mathbf{L} = 1)$  supermultiplets in 1967.[24] At the International Conference on High Energy Physics (the Rochester Conference) in Vienna in 1968, Haim Harari was the first rapporteur to suggest that the symmetric quark model is the correct model for baryons. The symmetric quark model was developed further by Alvaro De Rujula, Howard Georgi and Sheldon Glashow[25] in the context of the Coulomb potential suggested by quantum chromodynamics, by Dalitz and collaborators, and especially by Nathan Isgur and Karl[26]. Elizabeth Jenkins[27] gave a review of baryon spectroscopy from the standpoint of large  $N$  quantum chromodynamics. There is surprising similarity between the old results of Resnikoff and myself and the large  $N$  results.

Nambu was the pioneer in introducing three integer charged triplets and in having a gauge interaction that couples to the new three-valued degree of freedom via an octet of what we now call gluons. The proposal to have three triplets first appears in [19]. The coupling to the three-valued degree of freedom is in [20] and is developed further in [21]. Nambu and Han gave further analysis in [22].

Quantum chromodynamics has two facets; the hidden three-valued degree of freedom and the local  $SU(3)$  gauge theory with a vector octet of self-interacting

gluons that mediate the forces between the particles that carry the degree of freedom. These two facets are analogous to the electric charge as a degree of freedom and to the  $U(1)$  gauge theory in which photons mediate the interaction between charged particles in electromagnetism. The three-valued degree of freedom is implicit in the paraquark model, which is equivalent to color as a classification symmetry. The second facet of quantum chromodynamics is the local  $SU(3)$  color gauge interaction that provides the octet of gluons that mediate the force between color-carrying particles. These two facets taken together constitute quantum chromodynamics.

The word “color” that is colloquially attached to this new degree of freedom was introduced first by Pais[28] in a lecture at the Erice Summer School in 1965. Donald B. Lichtenberg[29] used the word independently in his book in 1970. Color became used generally after the papers of Gell-Mann[30] and of William A. Bardeen, Harald Fritzsch and Gell-Mann[31].

Some of the effects connected with color come from color as a classification symmetry. These include the classification of states of baryons and of mesons according to their constituent quark structure, the  $\pi^0 \rightarrow \gamma\gamma$  decay rate that follows from the axial anomaly, and the ratio of the electron-positron annihilation cross section to hadrons to the corresponding cross section to muon pairs. In each case, the number of quarks circulating in the quark loop gives the necessary factor  $N$  to agree with experiment. This  $N$  is equivalently either the number of Green components in the paraquark theory or the number of colors in the  $SU(3)$  color theory. Other effects require the gauged theory of color, including asymptotic freedom which allows the reconciliation of the constituent quark model for static properties of hadrons with the parton model for high-energy collision processes, as well as the observed running of the strong coupling constant, confinement, and the observed two-gluon and three-gluon jets.

The acceptance of the quark model occurred over a period of years. In the period 1964-1966, the main evidence was the baryon spectra, the magnetic moment ratio  $\mu_p/\mu_n$ , the Zweig rule, relations among cross sections, such as  $\sigma_{\pi N}/\sigma_{NN} = 2/3$  that follow from simple quark counting arguments. Later the  $\pi^0$  decay and the electron-positron annihilation cross section ratio provided support. In 1969 the SLAC deep inelastic electron scattering experiments as interpreted using Bjorken

scaling, Feynman's parton model, and the Bjorken-Paschos quark-parton model greatly strengthened the case. However only in 1974, with the discovery of the  $J/\Psi$  and its friends as bound states of charm and anticharm quarks, did the quark model with color become accepted generally.

Quantum chromodynamics can be summarized in terms of its gauge Lagrangian for  $SU(3)$  color with three generations of color-triplet quark matter fields. The reason for three generations is beyond the scope of quantum chromodynamics and is not yet known. The main features of quantum chromodynamics as a gauge theory are the running of the coupling constant and the associated asymptotic freedom at high energy and infrared slavery which leads to permanent confinement of color-carrying particles (at zero temperature).

QCD has passed many tests, including the running of  $\alpha_{strong}$ , the jets in hadronic collisions, the (modified) scaling of scattering processes, and the parton model structure and fragmentation functions. QCD agrees well with data on heavy quarkonium decays, electron-positron annihilation to hadrons and to leptons, and with the measurement of the  $Z$  width, which gives a restriction on the number of neutrinos with mass below half the mass of the  $Z$ .

I conclude with cursory remarks about the theoretical analysis of QCD. Broadly speaking, there are three approaches, (1) models that don't try to start from first principles, i.e. from the QCD Lagrangian. There are many models, none universally accepted, (2) continuum methods, mainly using Bethe-Salpeter methods, which have had, at best, limited success, and (3) lattice QCD, pioneered by Kenneth Wilson, which at present seems to be the most productive approach, and, at least, has stimulated development of supercomputers.

This whole story of the progress from Wigner's supermultiplet theory of nuclei to quantum chromodynamics illustrates Wigner's profound influence on the physics of his time and beyond: the centrality of symmetry principles, the use of deep general mathematical results, and the direct contact with physical phenomena.

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