Confronting the QCD photon structure with recent data

Jiří Chýla

Center for Particle Physics, Institute of Physics, Prague

- **Concept of photon structure**
- **Basic facts and formulae**
- **Semantics: plea for a common language**
- **PDF of the photon**
- **QCD effects in data on interactions of quasireal photons**
- **Virtual photon structure: why**
- **QCD effects in data on interactions of virtual photons?**
- **Resolved longitudinal photon: why not?**

Details in **JHEP04(2000)007; PRD 62 (2000), 114025; PL B488 (2000), 289; EPJ C16 (2000), 471, EPJ C18 (2001), 723**

Concept of photon structure

All the present knowledge of the structure of the photon comes from experiments at the ep and e^+e^- colliders, where the incoming leptons act as sources of transverse and longitudinal virtual photons

$$f_T^\gamma(y,P^2) = \frac{\alpha}{2\pi}\left(\frac{1+(1-y)^2}{y}\frac{1}{P^2} - \frac{2m_e^2 y}{P^4}\right),$$

$$f_L^\gamma(y,P^2) = \frac{\alpha}{2\pi}\frac{2(1-y)}{y}\frac{1}{P^2}.$$

(Quasi)real photon: $P^2 \ll \Lambda_{\mathrm{QCD}}$

Small masses of light quarks implies nonperturbative QCD effects which **necessitate the introduction of the concept of PDF of the photon.** Expected to have (almost) the **same properties** as truly real photon.

Virtual photon: smooth transition from the essentially nonperturbative to the perturbative region for $P^2 \gg \Lambda_{\mathrm{QCD}}$, in practice $P^2 \gtrsim 2$ GeV^2.

Basic facts and formulae

PDF of the photon satisfy the system of **inhomogeneous** evolution equations

$$\begin{aligned}\frac{\mathrm{d}\Sigma(x,M)}{\mathrm{d}\ln M^2} &= \delta_\Sigma k_q + P_{qq}\otimes\Sigma + P_{qG}\otimes G,\\ \frac{\mathrm{d}G(x,M)}{\mathrm{d}\ln M^2} &= k_G + P_{Gq}\otimes\Sigma + P_{GG}\otimes G,\\ \frac{\mathrm{d}q_{\mathrm{NS}}(x,M)}{\mathrm{d}\ln M^2} &= \delta_{\mathrm{NS}} k_q + P_{\mathrm{NS}}\otimes q_{\mathrm{NS}},\end{aligned}$$

where $\delta_{\mathrm{NS}} \equiv 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right)$, $\delta_\Sigma = 6n_f\langle e^2\rangle$ and

$$\begin{aligned} k_q(x,M) &= \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right],\\ k_G(x,M) &= \frac{\alpha}{2\pi}\left[\frac{\alpha_s(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right],\\ P_{ij}(x,M) &= \frac{\alpha_s(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots.\end{aligned}$$

$P_{ij}^{(0)}, k_q^{(0)}, k_G^{(0)}$ are **unique** but all higher order splitting functions **arbitrary**.

The structure function $F_2^\gamma(x,Q^2)$ is given as

$$\frac{1}{x}F_2^\gamma(x,Q^2) = q_{\rm NS}(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}\delta_{\rm NS}C_\gamma +$$
$$\langle e^2\rangle\Sigma(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}\langle e^2\rangle\delta_\Sigma C_\gamma+\langle e^2\rangle G(M)\otimes C_G(Q/M),$$

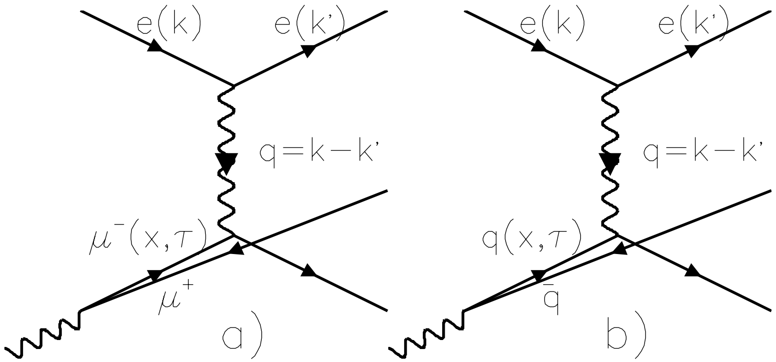
where C_q, C_G, C_γ can be expanded in powers of $\alpha_s(\mu)$

$$\begin{aligned}
C_q(x,Q/M) &= \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(x,Q/M)+\cdots,\\
C_G(x,Q/M) &= \frac{\alpha_s(\mu)}{2\pi}C_G^{(1)}(x,Q/M)+\cdots,\\
C_\gamma(x,Q/M) &= C_\gamma^{(0)}(x,Q/M)+\frac{\alpha_s(\mu)}{2\pi}C_\gamma^{(1)}(x,Q/M)+\cdots,\\
C_\gamma^{(0)}(x,Q/M) &= (x^2+(1-x)^2)\ln\frac{Q^2(1-x)}{M^2x}+\kappa(x)
\end{aligned}$$

where $\kappa(x)\equiv 8x(1-x)-1$. C_γ^0 as well as $k_q^{(0)}=(x^2+(1-x)^2)$ **come from pure QED**, which provides the lowest order contribution to F_2^γ in the form

$$\frac{1}{x}F_2^{\gamma,{\rm QED}}(x,Q^2)=\sum_{i=1}^{n_f}e_i^2\left(q_i^{\rm QED}(x,Q)+\overline{q}_i^{\rm QED}(x,Q)\right)+\frac{\alpha}{2\pi}6n_f\langle e^4\rangle C_\gamma^{(0)}(x,1)$$

QED distributions functions of the real photon



$$\frac{\mathrm{d}\sigma(e^-\gamma\to e^-q\overline{q})}{\mathrm{d}x\mathrm{d}Q^2}=\frac{2\pi\alpha^2}{xQ^4}F_{2,q}^{\gamma}(x,Q^2)\left(1+(1-y)^2\right)$$

where, denoting $\kappa(x)=8x(1-x)-1$,

$$F_{2,q}^{\gamma}(x,Q^2)=\frac{\alpha}{2\pi}2e_l^4x\left[k_q^{(0)}(x)\ln\frac{Q^2(1-x)}{m_q^2x}+\kappa(x)\right]$$

can be separated it into two parts

$$F_{2,q}^{\gamma,\mathrm{dir}}(x,Q^2)\equiv\frac{\alpha}{2\pi}6e_q^4x\left[k_q^{(0)}\ln\frac{Q^2}{M^2x}+6x(1-x)-1\right],$$

$$F_{2,q}^{\gamma,\mathrm{res}}(x,Q^2)\equiv\frac{\alpha}{2\pi}6e_q^4x\left[k_q^{(0)}(x)\ln\frac{M^2(1-x)}{m_q^2}+2x(1-x)\right],$$

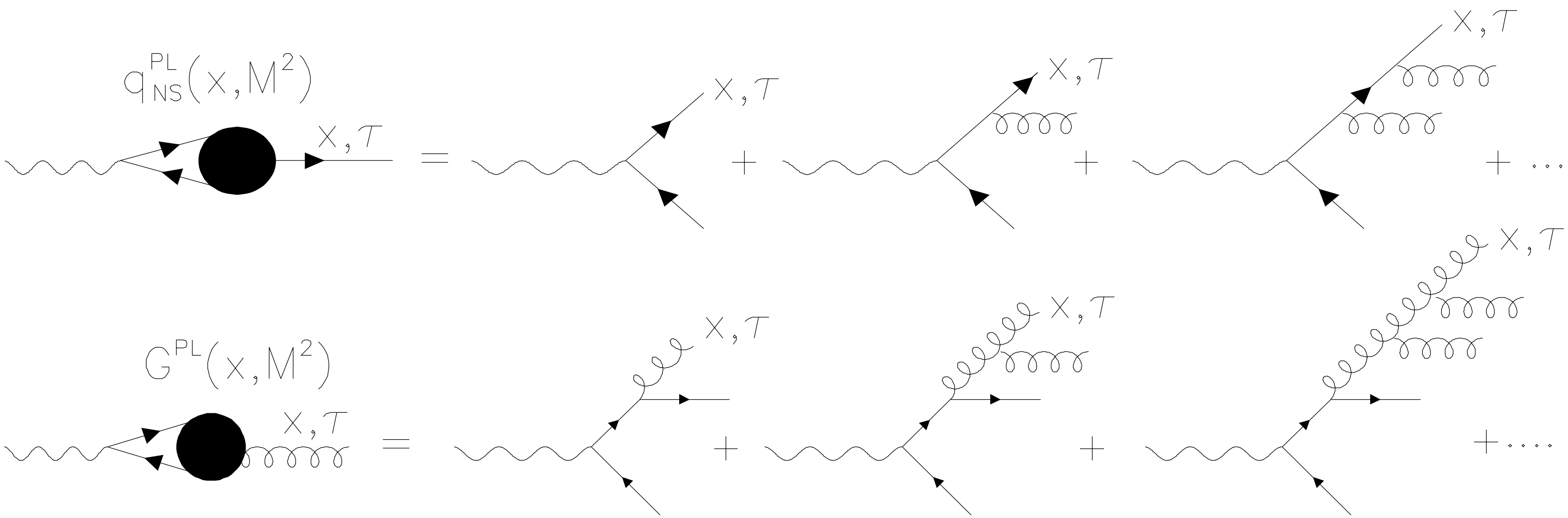
the latter coming from region of almost collinear $\gamma\to q\overline{q}$ splitting and defining the **QED parts** of quark and gluon distribution functions of the photon

$$q^{\mathrm{QED}}(x,M)\equiv\frac{\alpha}{2\pi}3e_q^2k_q^{(0)}(x)\ln\frac{M^2(1-x)}{m_q^2}+2x(1-x),\quad G(x,M)\equiv 0.$$

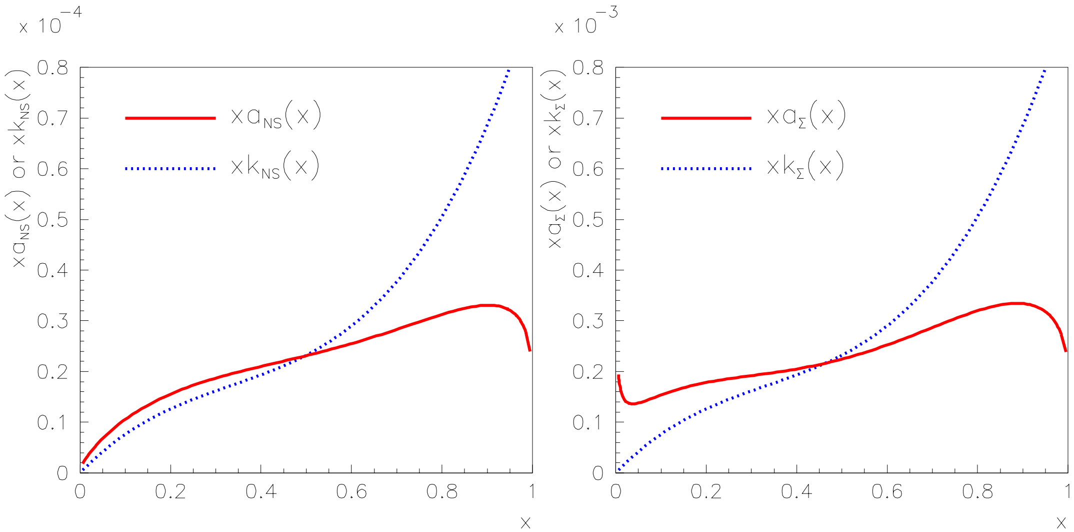
The presence of the inhomogeneous terms implies that their general solutions can be written as a sum

$$D(x,M) = D^{\mathrm{PL}}(x,M,M_0) + D^{\mathrm{HAD}}(x,M,M_0).$$

of a particular solution of the full inhomogeneous equations and a general solution, called **hadron-like** (HAD), of the corresponding homogeneous ones. A subset of the former resulting from the resummation of contributions of diagrams describing multiple parton emissions off the primary QED vertex $\gamma \to q\overline{q}$ and vanishing at $M = M_0$, are called **point-like** (PL).



$$q_{\rm NS}^{\rm PL}(n,M_0,M)=\frac{4\pi}{\alpha_s(M)}\left[1-\left(\frac{\alpha_s(M)}{\alpha_s(M_0)}\right)^{1-2P_{qq}^{(0)}(n)/\beta_0}\right]a_{\rm NS}(n)$$



$$a_{\rm NS}(n)\equiv\frac{\alpha}{2\pi\beta_0}\frac{k_{\rm NS}^{(0)}(n)}{1-2P_{qq}^{(0)}(n)/\beta_0}$$

where the presence of α_s in the denominator is often interpreted as evidence that

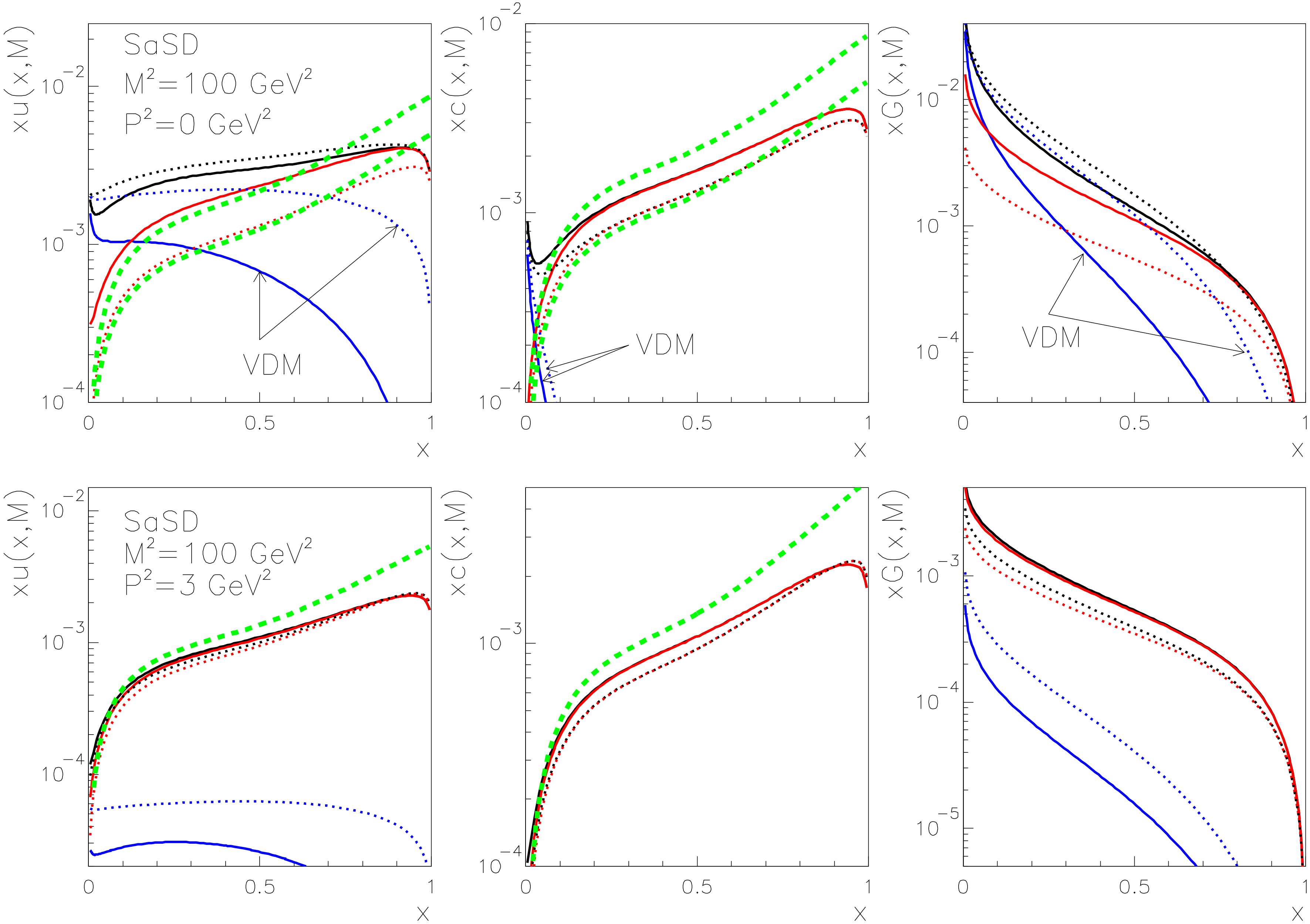
$$q_{\rm NS}^{\rm PL}(n,M_0,M)\propto\frac{\alpha}{\alpha_s}$$

but this is untenable because **switching QCD off** by sending $\Lambda_{\rm RS}\rightarrow 0$ for fixed M, M_0 reduces the above expression to the **purely QED contribution**

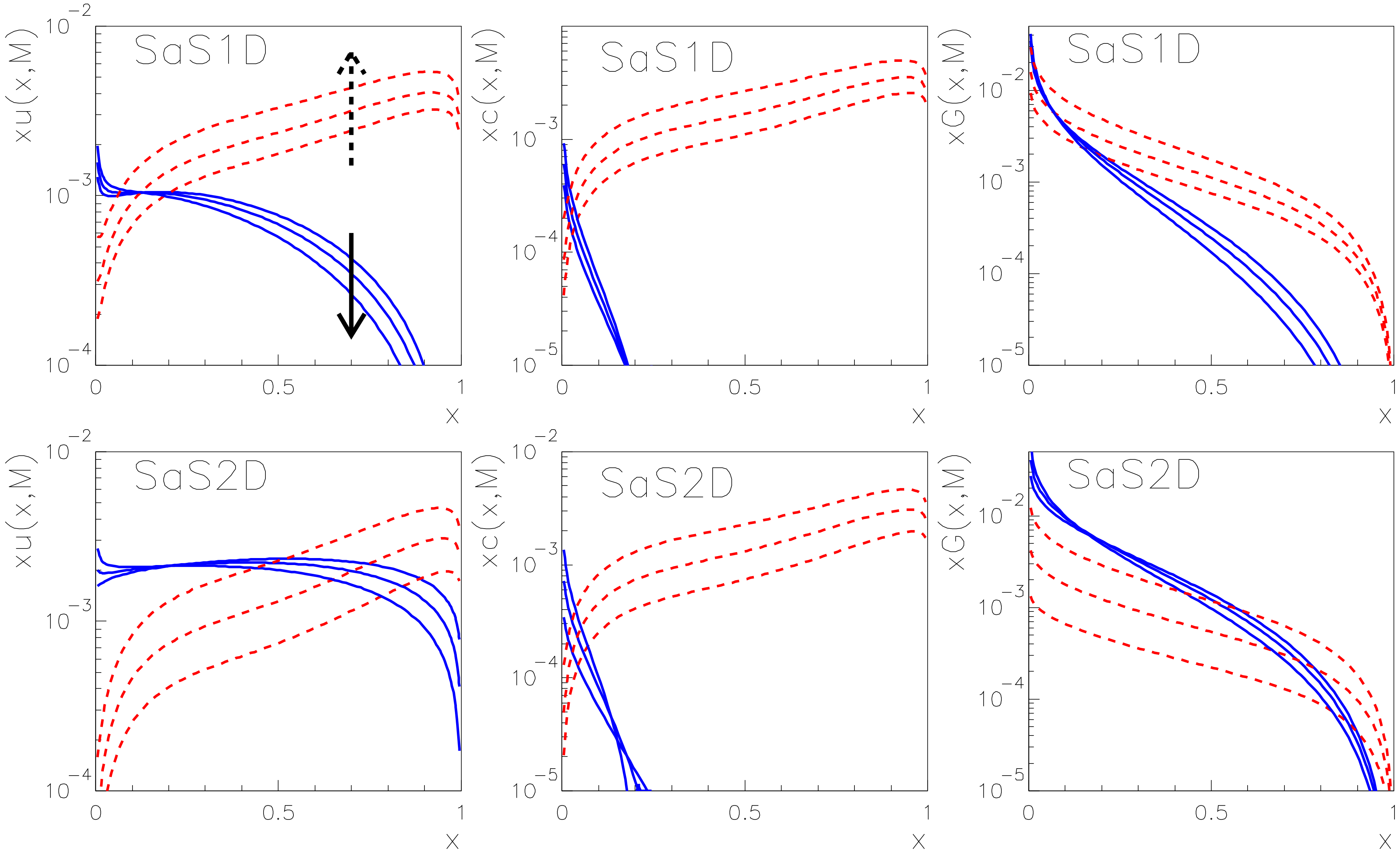
$$q_{\rm NS}^{\rm PL}(x,M,M_0)\rightarrow q_{\rm NS}^{\rm QED}(x,M,M_0)=\frac{\alpha}{2\pi}k_{\rm NS}^{(0)}(x)\ln\frac{M^2}{M_0^2}.$$

Basic features of hadronlike and pointlike parts of photonic PDFs

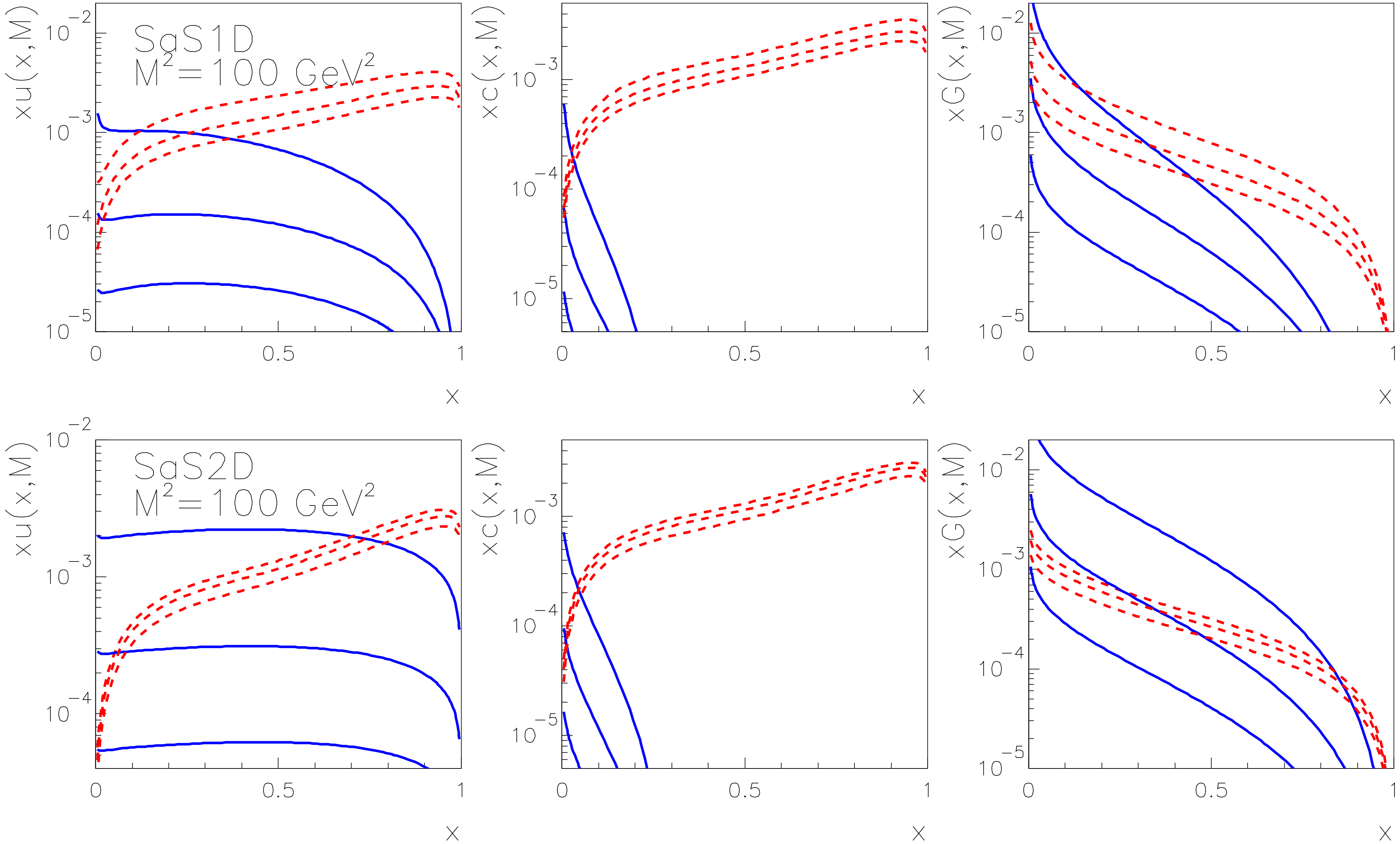
- The separation inherently **ambiguous**.
- Only **their sum relevant** for calculation of cross sections of hard processes
- Separation **important for multiple interactions** (PYTHIA).
- Both parts describe **QCD effects**,
- but exhibit different **factorization scale** dependence:
  - hadron-like part **entirely due to QCD** effects, whereas
  - point-like part **dominated by QED** splitting $\gamma \to q\overline{q}$.
- as well as **virtuality** dependence
  - pointlike: slow **logarithmic** decrease
  - hadronlike: fast **powerlike** decrease
- Both
  - **generate gluons**
  - **rise at low** x


SaSD
M²=100 GeV²
P²=0 GeV²
VDM
VDM
VDM
SaSD
M²=100 GeV²
P²=3 GeV²
xu(x,M)
xc(x,M)
xG(x,M)
x

Factorization scale dependence



Virtuality dependence



For $P^2 \gtrsim 2$ GeV^2 only the pointlike parts of PDF relevant!

Semantics: plea for a common language

Agreement on semantics is a prerequisite for any meaningful discussion of the photon structure. I prefer the terminology advocated by GRV:

- **Direct & resolved** photon
- **Pointlike & hadronlike** contributions
- **QED & QPM** contributions

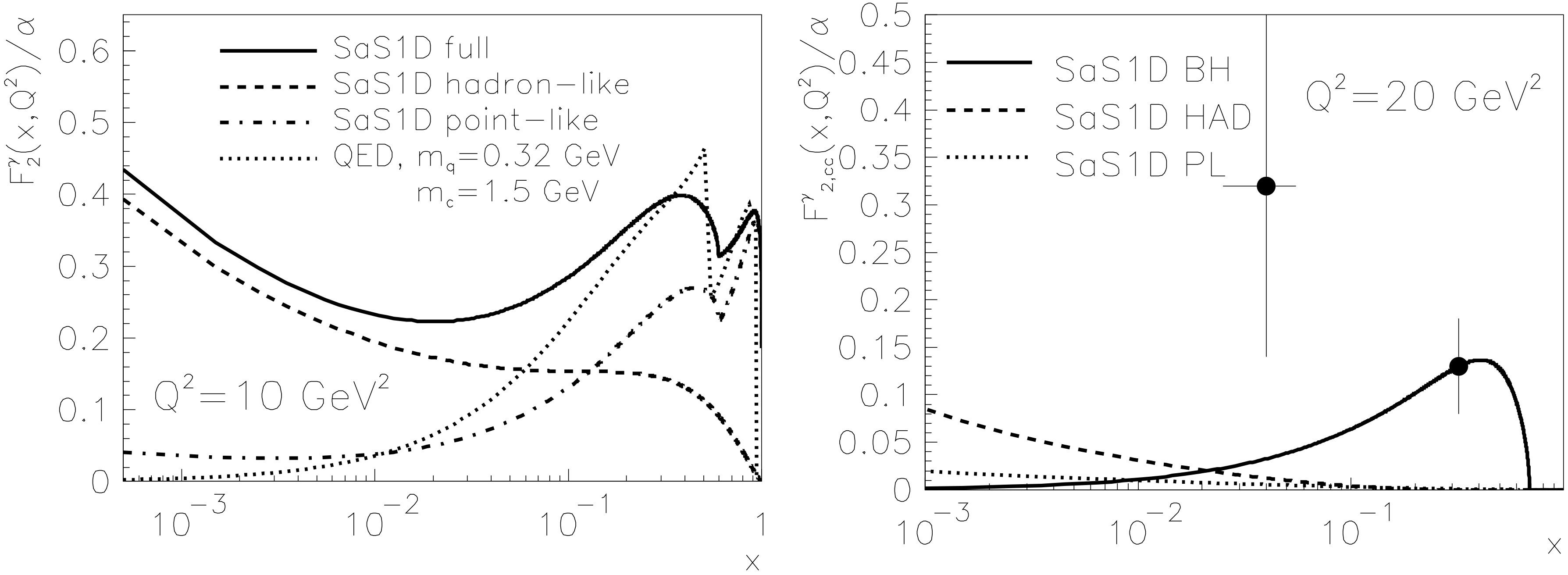
Unfortunately, **different names are used for the same content**

- **Bare** photon instead of **direct** photon contribution
- **Anomalous** part instead of **pointlike** part of photonic PDF
- **VMD** part instead of **hadronic** part of photonic PDF

and even worse, **one notion is used in different meanings**

- **Pointlike and hadronlike** instead of **direct and resolved** contribution
- **LO and NLO** interpreted differently than for $R_{e^+e^-}$

Example of resulting confusion: **OPAL** papers on dijets, F_2^γ and F_{2c}^γ:



dijets [EPJC 10 (1999), 547]: see just **resolved contribution** but says

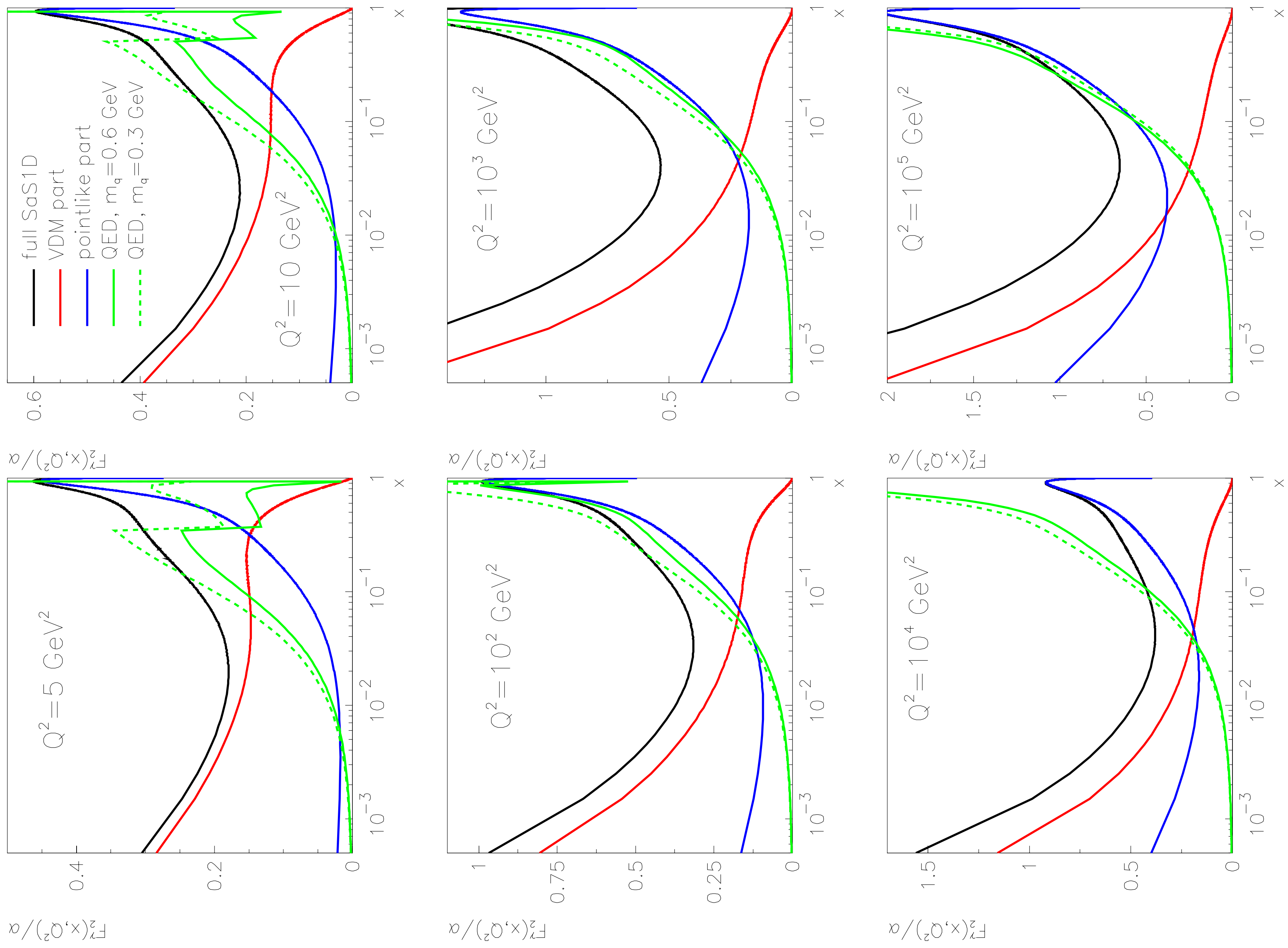
"photons appear resolved through its fluctuations into hadronic components"

$\mathbf{F_2^\gamma}$ [EPJC 18 (2000), 15]: see dominant **hadron-like contribution** but claims

"photon must contain a significant hadron-like component at low x."

$\mathbf{F_{2c}^\gamma}$ [EPJC 16 (2000), 579]: see **excess** over hadron-like contribution but claims

"the measurement suggests a nonzero hadron-like component of $F^\gamma_{2,c\bar{c}}$"


full SaS1D
VDM part
pointlike part
QED, m_q=0.6 GeV
QED, m_q=0.3 GeV
Q^2=5 GeV^2
Q^2=10 GeV^2
Q^2=10^2 GeV^2
Q^2=10^3 GeV^2
Q^2=10^4 GeV^2
Q^2=10^5 GeV^2
$F_2^\gamma(x,Q^2)/\alpha$
x

PDF of the photon

Transverse photon

Glück, Reya, Vogt (1992): $P^2 = 0$, HAD+PL

Glück, Reya, Stratmann (1995): $P^2 \lesssim M^2/5$, HAD+PL

Schuler, Sjöstrand (1995): $P^2 \lesssim M^2$, HAD, PL separately

Glück, Reya, Schienbein (1999): improved GRS (1995)

other less often used: **AFG, WHIT, GS**

new: Cornet, Jankowski, Krawczyk, Lorca (2003): $P^2 = 0$, heavy quarks

Longitudinal photon

Friberg, Sjöstrand (2000): rescaled $D_p^{\gamma_T}$

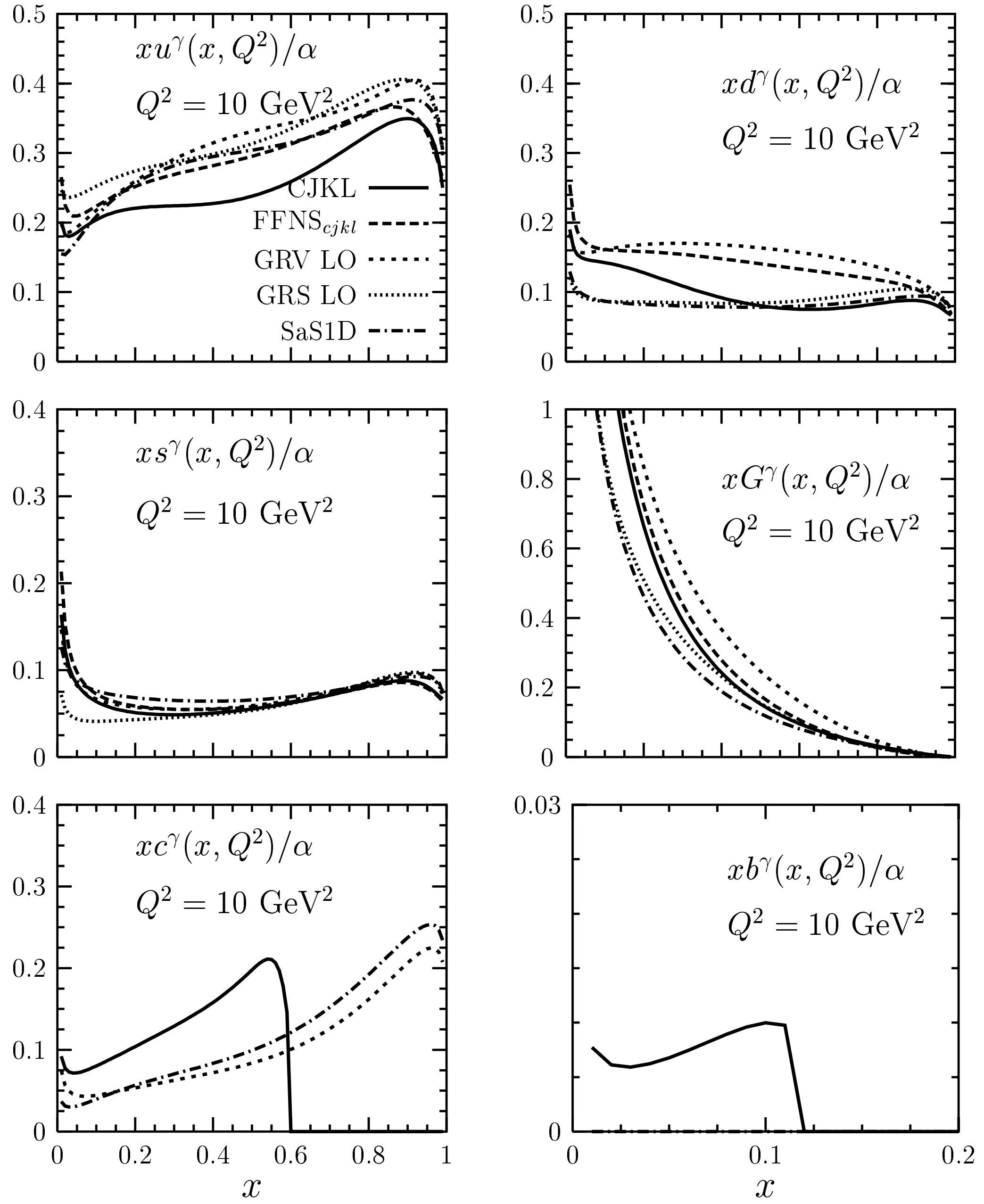
Chýla (2000): LO QCD evolution "dynamically" generated from QED contribution, $P^2 \ll M^2$.

QCD effects in interactions of quasireal photons

- Scaling violations in $F_2^\gamma(x, Q^2)$
- Magnitude of $F_2^\gamma(x, Q^2)$ at low x
- Jets in γp collisions
- Jets in $\gamma\gamma$ collisions
- Heavy quarks in γp collisions
- Heavy quarks in $\gamma\gamma$ collisions

Fits to PDF of the photon

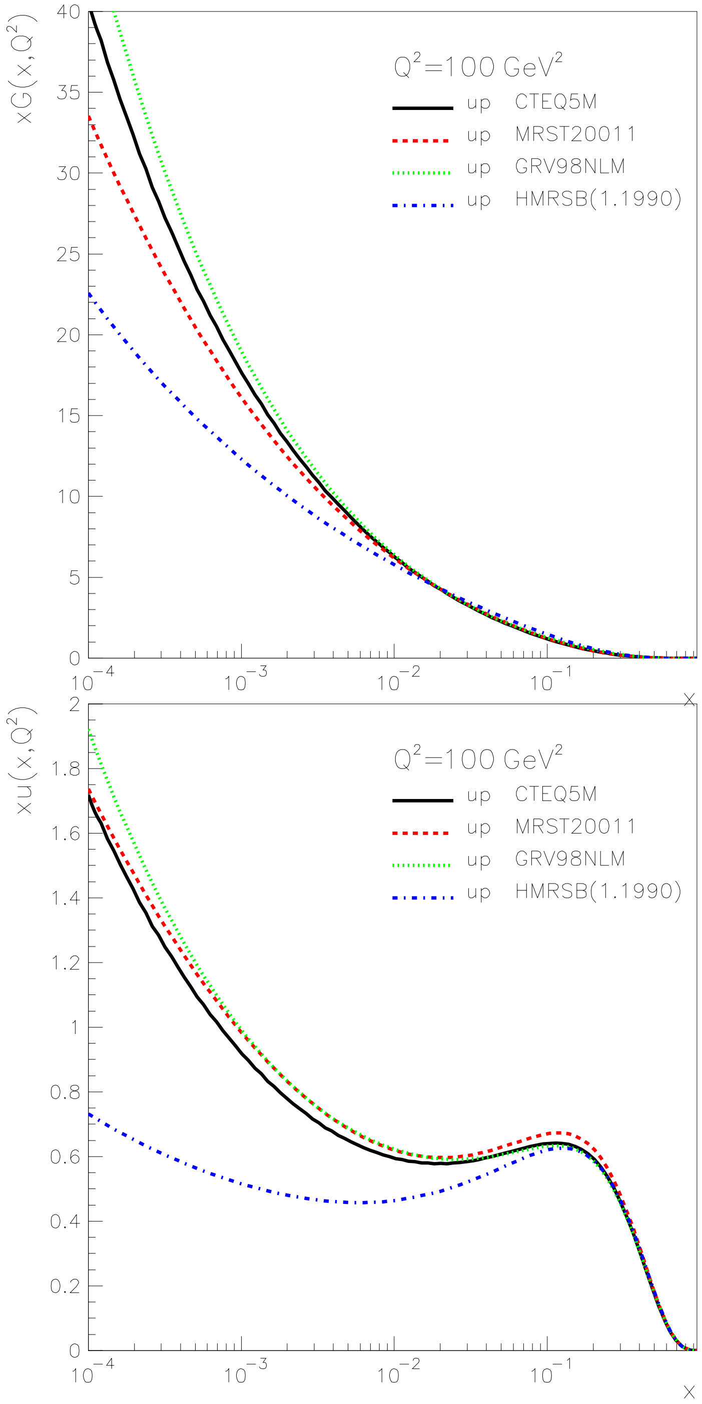
model	N	# of data points			
		205		182 - no TPC	
		χ^2	χ^2/DOF	χ^2	χ^2/DOF
SaS1D	6	657	3.30	611	3.47
GRS LO	0	499	2.43	366	2.01
FFNS_{CJKL}	3	442	2.19	357	1.99
CJKL	3	406	2.01	323	1.80



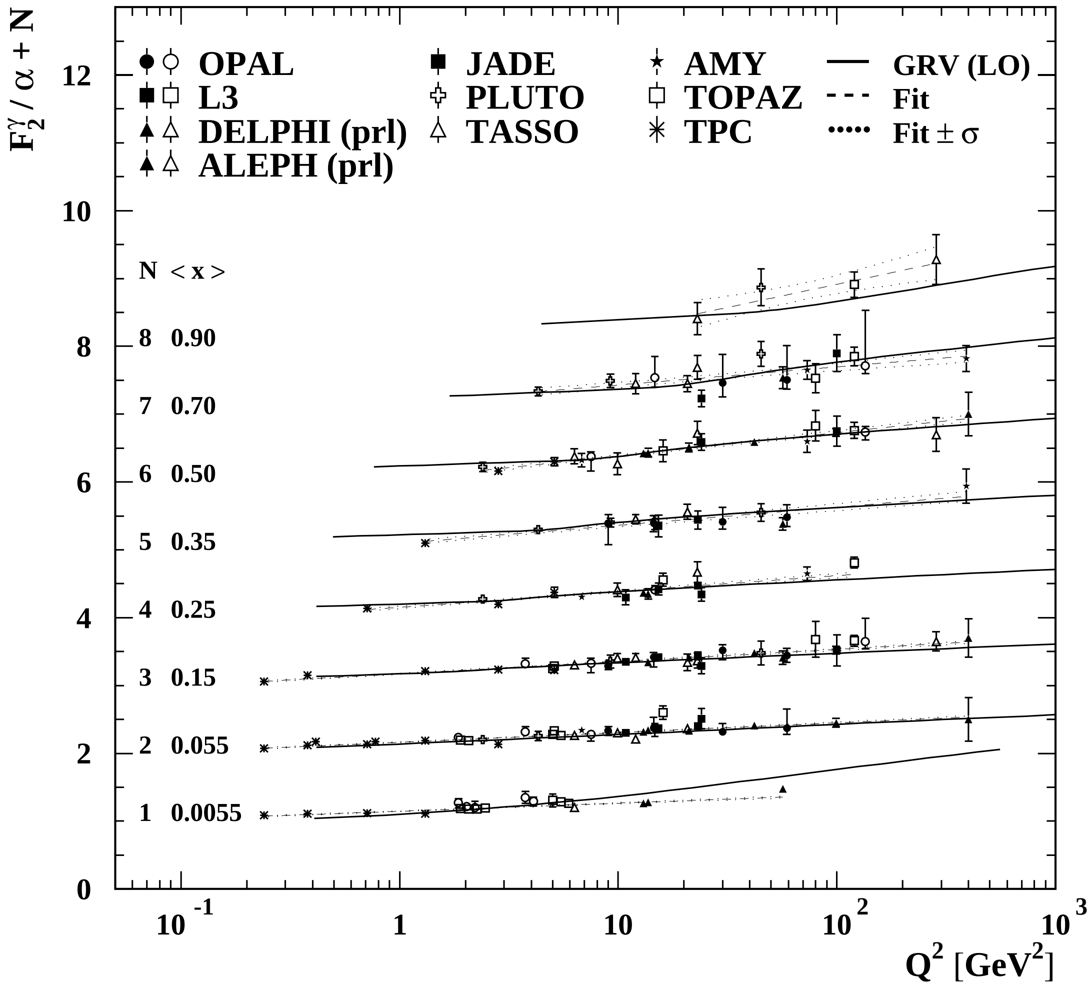
- Still **poor fit** and
- **large differences** between extracted PDF at moderate x

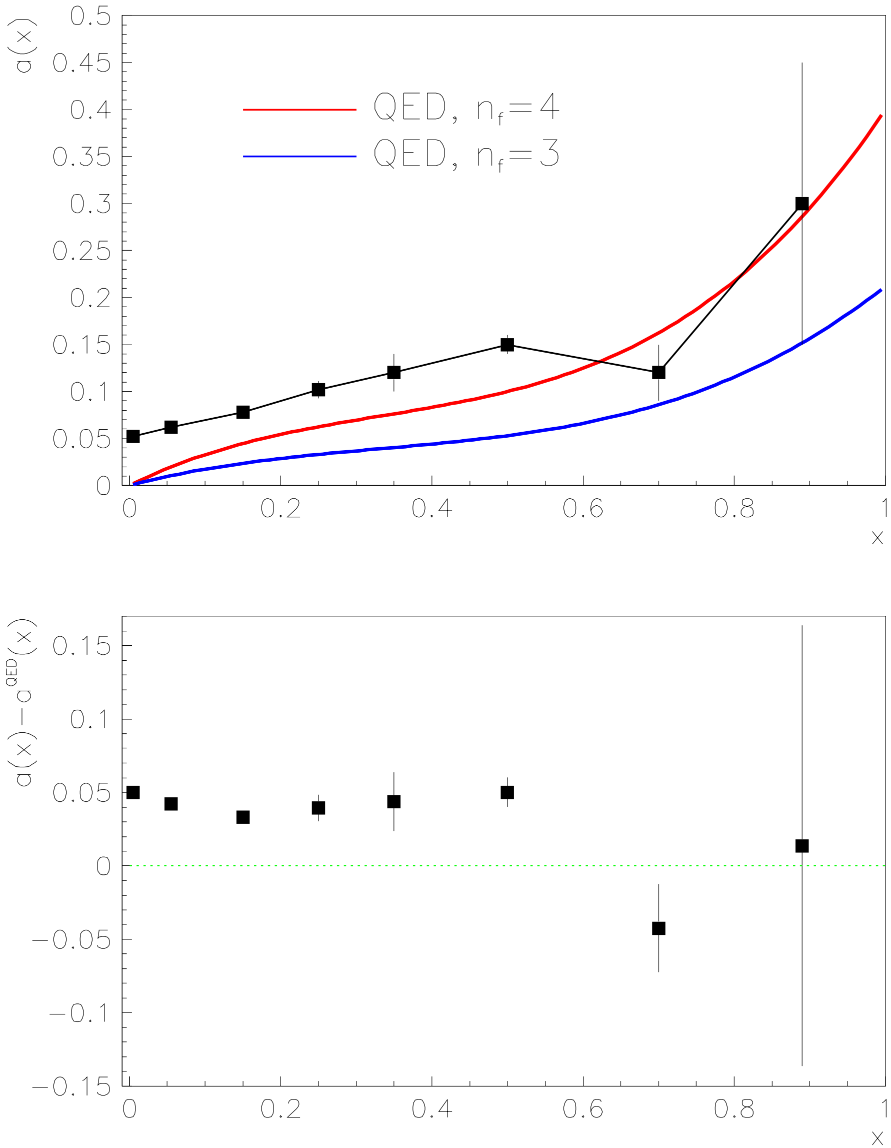
Fits to PDF of the proton

Data set	N	MRST	MRST 0.117	MRST 0.121	MRST J
H1 ep	400	382	386	378	377
ZEUS ep	272	254	255	258	253
BCDMS μp	167	193	182	208	183
BCDMS μd	155	218	211	226	219
NMC μp	126	134	143	127	135
NMC μd	126	100	108	95	100
SLAC ep	53	66	71	63	67
SLAC ed	54	56	67	47	58
E665 μp	53	51	50	52	51
E665 μd	53	61	61	61	61
CCFR $F_2^{\nu N}$	74	85	88	82	89
CCFR $F_3^{\nu N}$	105	107	103	112	110
NMC n/p	156	155	155	153	161
E605 DY	136	232	229	247	273
Tevatron Jets	113	170	168	167	118
Total	2097	2328	2346	2345	2337



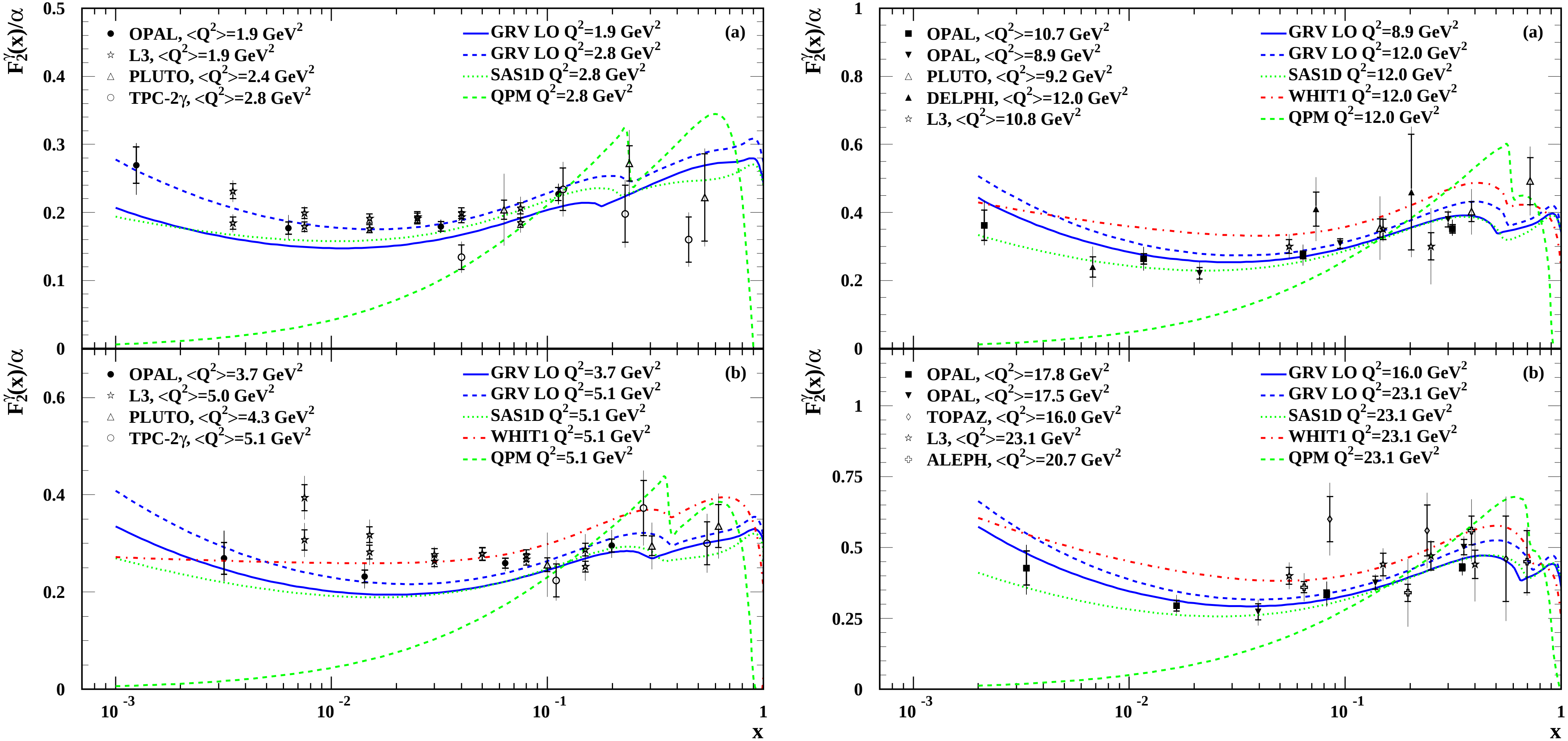
Scaling violations in F_2^γ



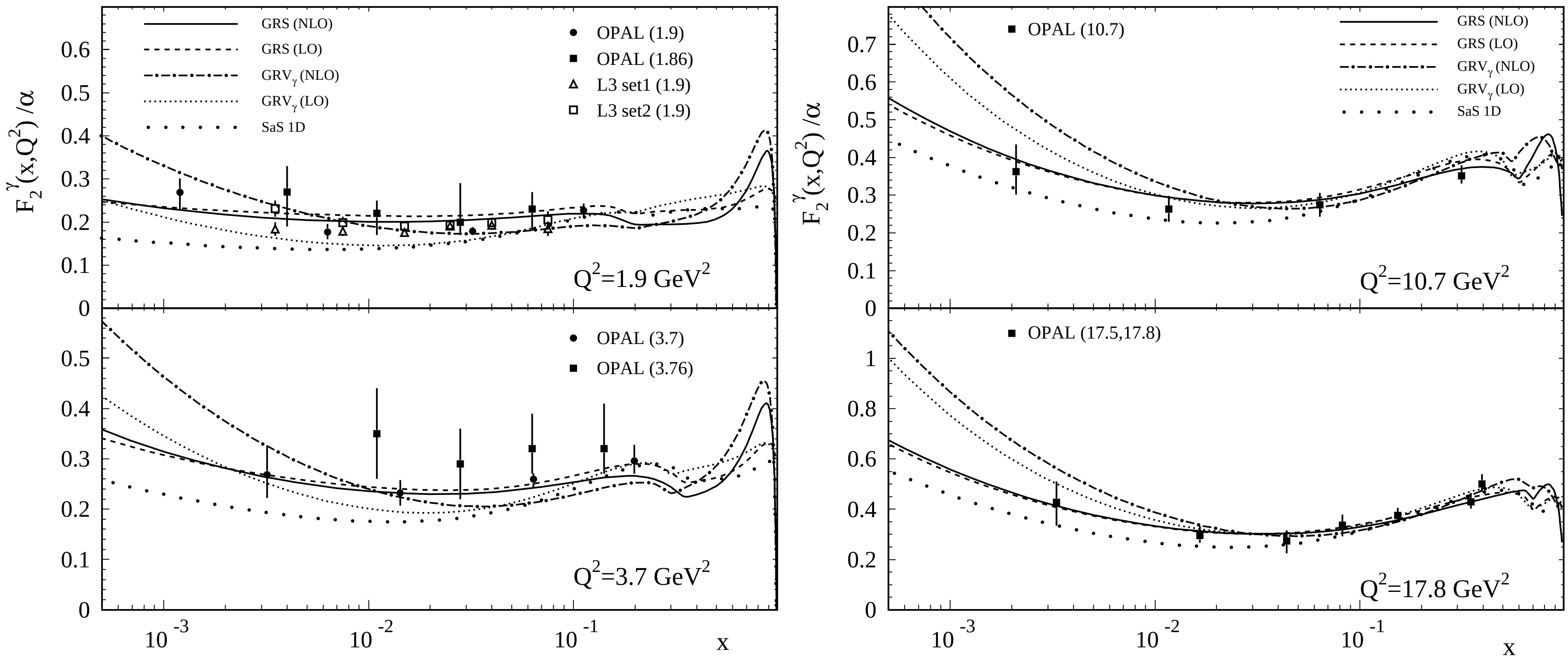


inconclusive!

$F_2^\gamma(x, Q^2)$ at low x



$F_2^\gamma(x, Q^2)$ at low x & new GRS parameterization



"small x measurements imply that photon must contain a dominant hadron-like component at low x"

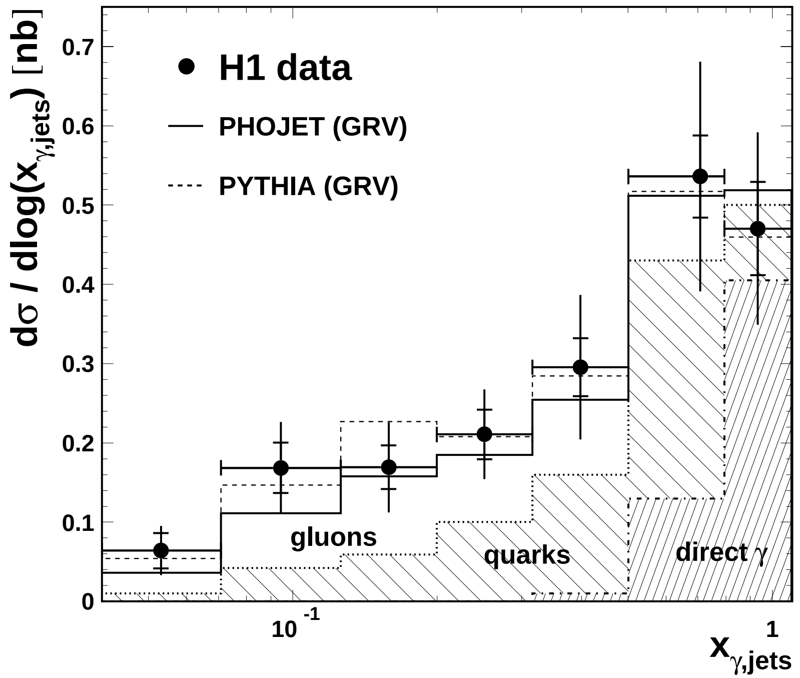
Jets in γp collisions

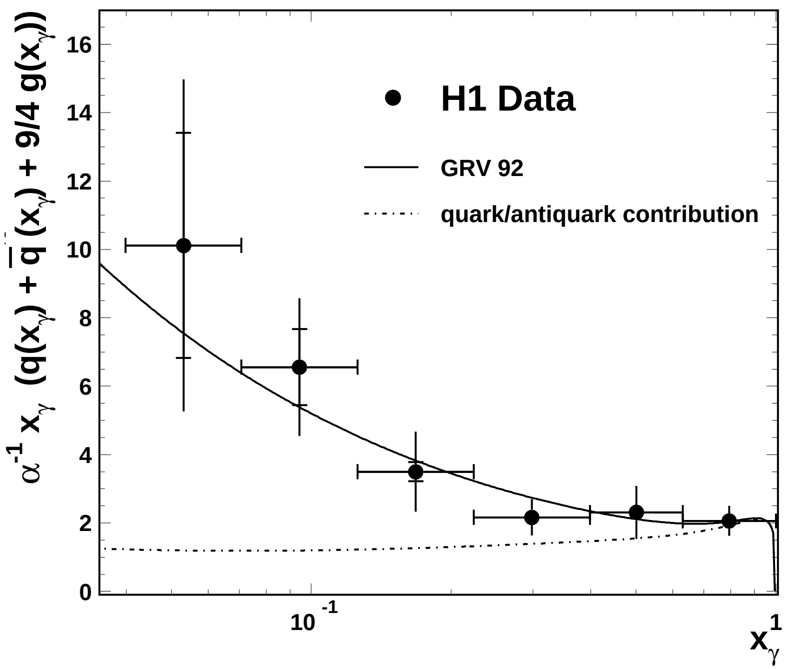
$$\frac{d^4\sigma^{ep}}{dydx_\gamma, dx_p, d\cos\Theta} =$$

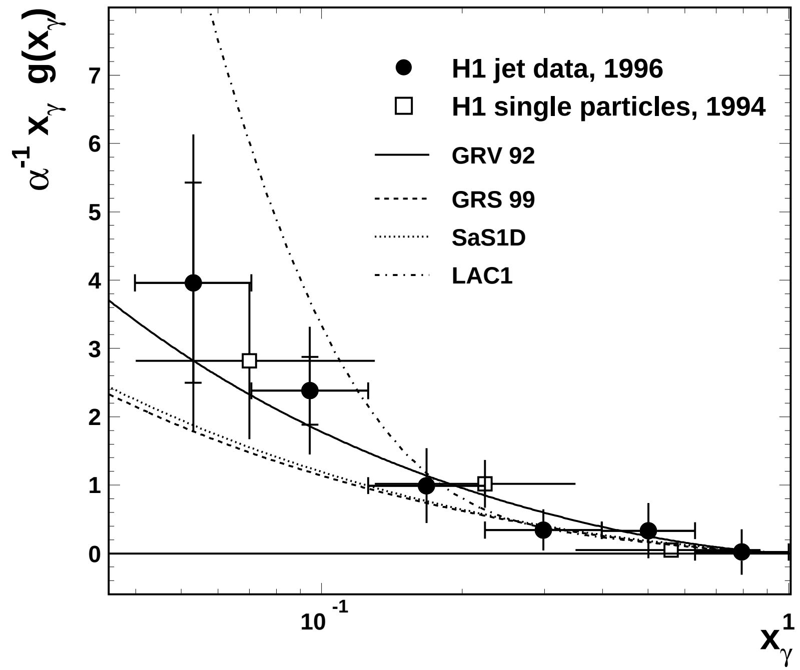
$$\frac{1}{32\pi s_{ep}}\,\frac{f_{\gamma/e}}{y}\,\frac{f_{\gamma,eff}(x_\gamma)f_{p,eff}(x_p)}{x_\gamma x_p}\,\frac{d\sigma}{d\cos\Theta}$$

$$f_{\gamma,eff}(x_\gamma) = [q(x_\gamma) + \overline{q}(x_\gamma) + (9/4)g(x_\gamma)]$$

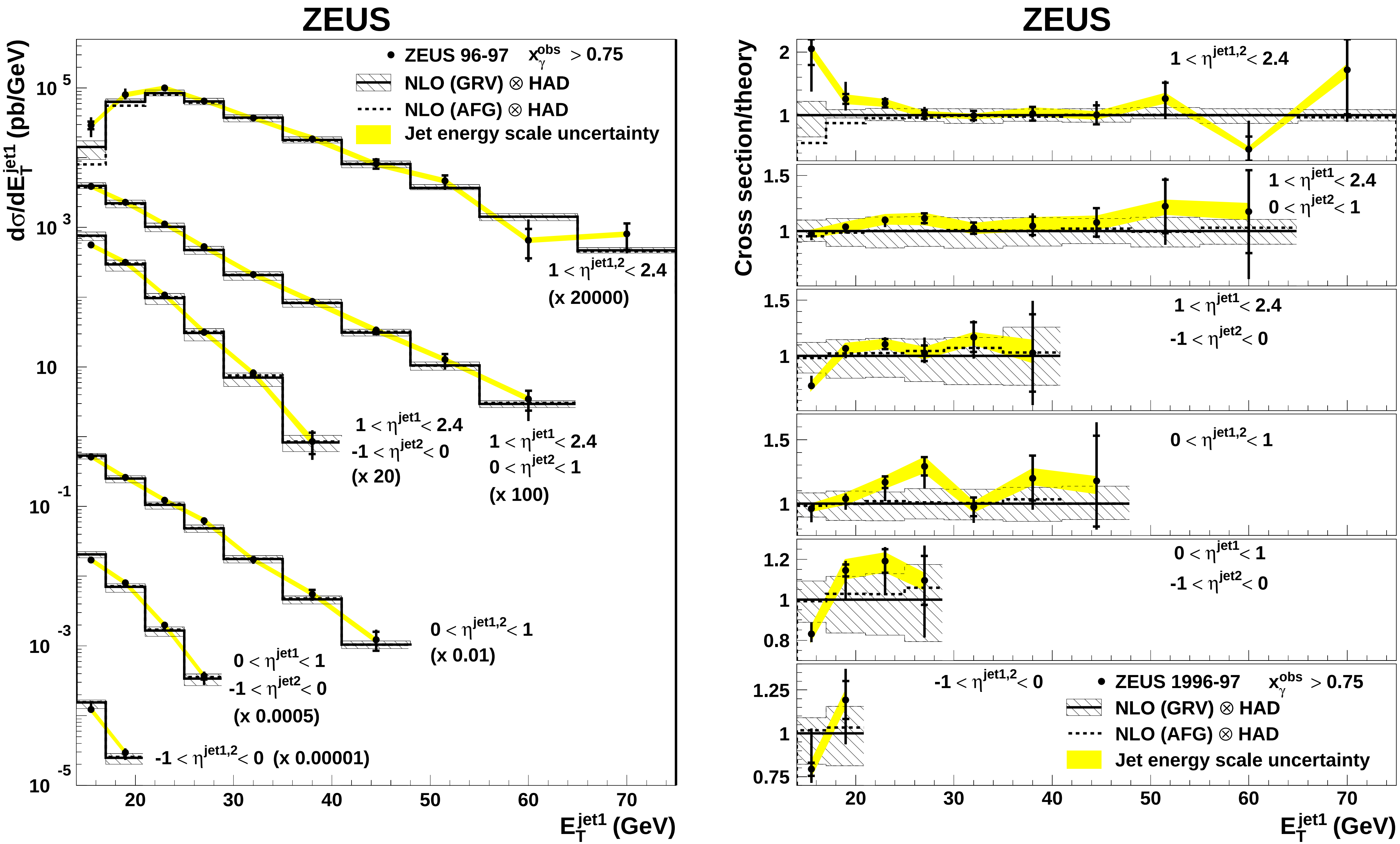
$$f_{p,eff}(x_p) = [q(x_p) + \overline{q}(x_p) + (9/4)g(x_p)]$$





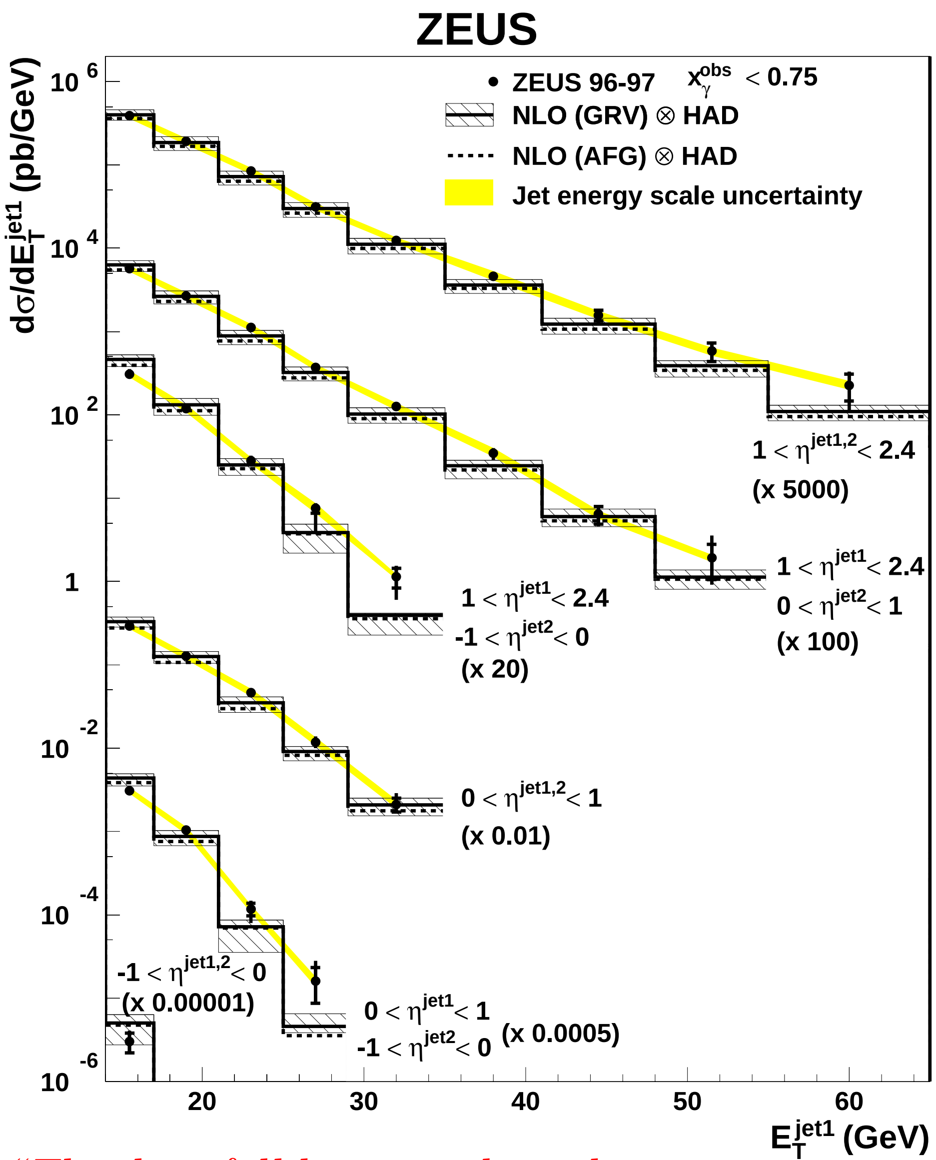


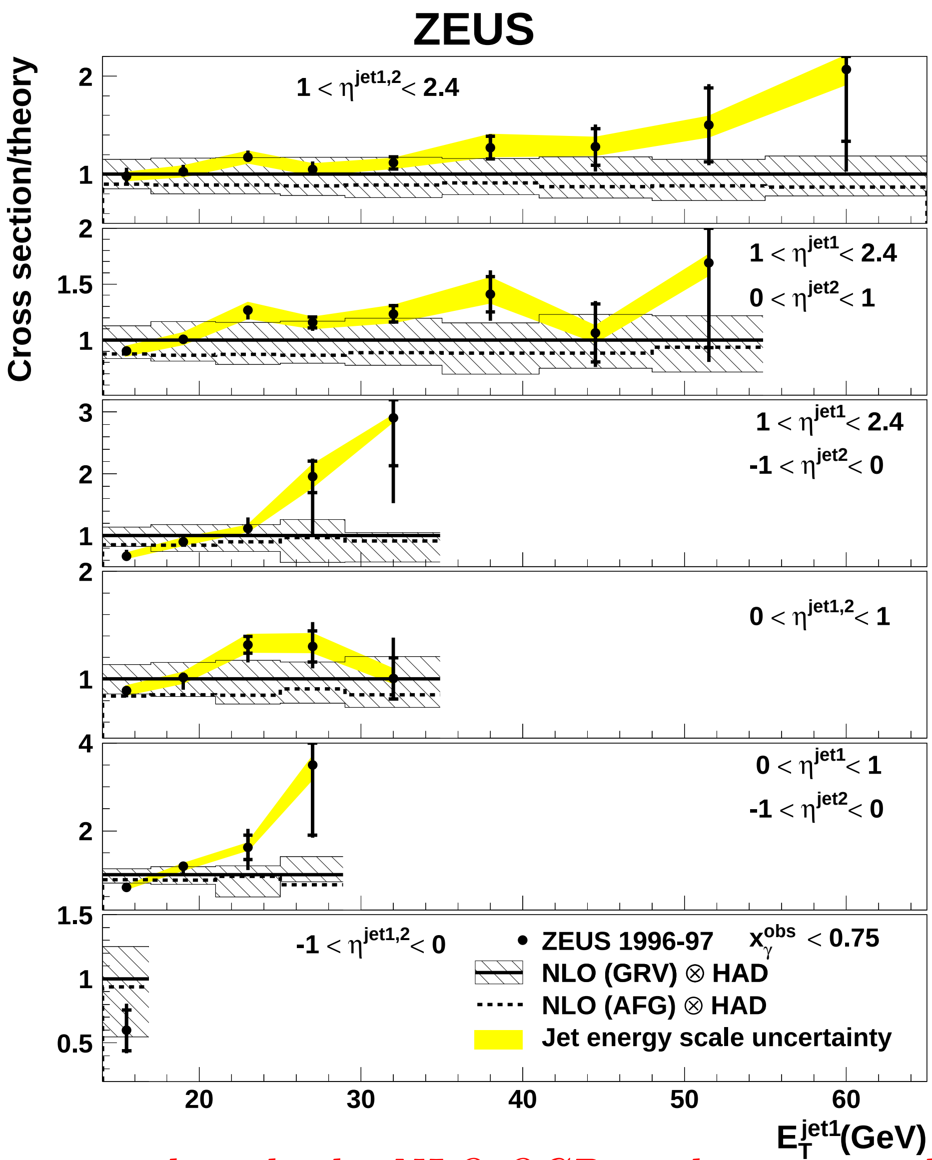
Jets in direct photoproduction



Data in very good agreement with predictions of perturbative QCD

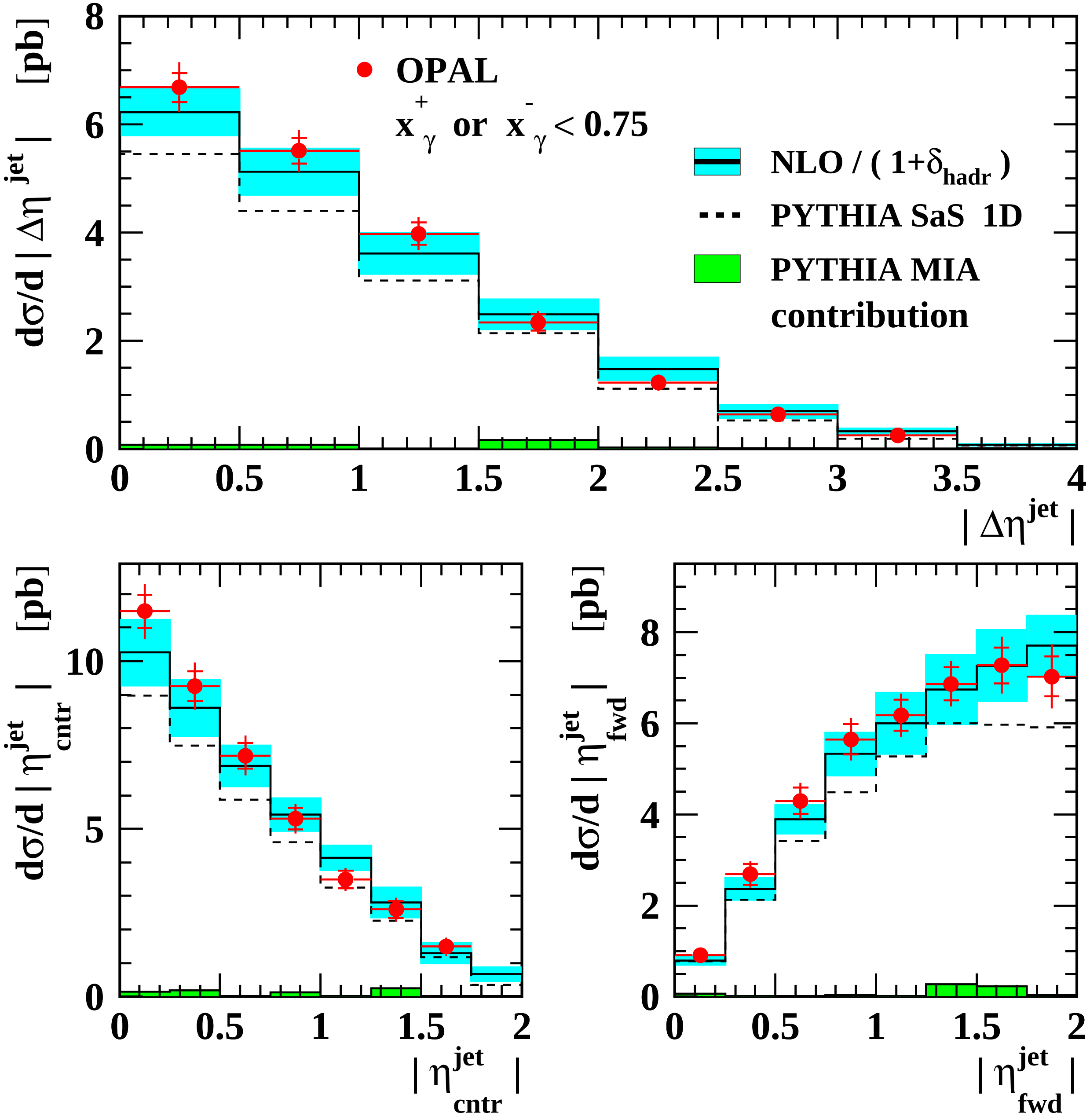
Jets in resolved photoproduction



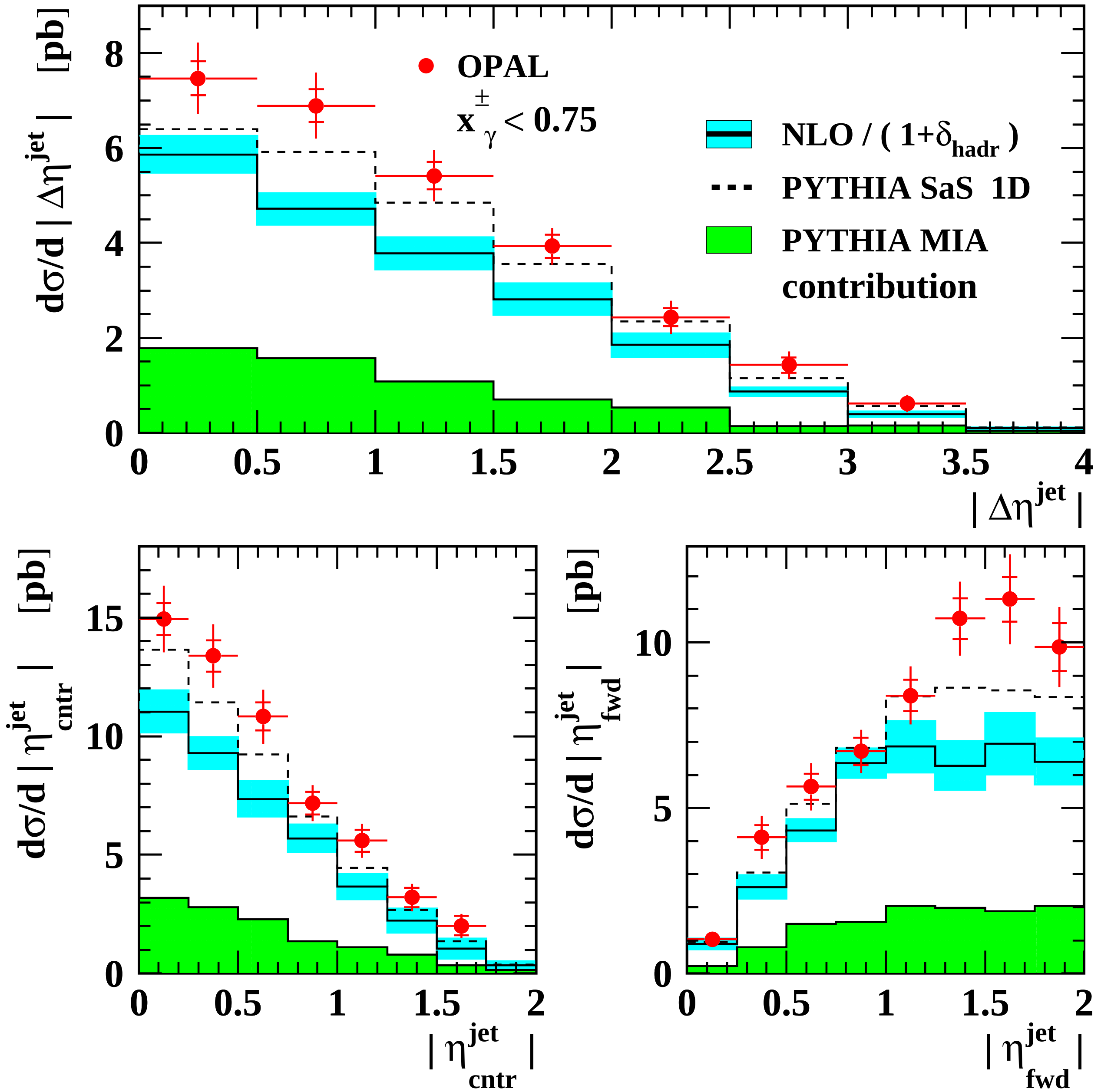


"The data fall less steeply with increasing transverse energy than do the NLO QCD predictions, and show sensitivity to the parton densities of the photon. Neither the AFG-HO nor the GRV-HO parameterizations convoluted with the NLO matrix elements fully describe all features of the data."

Jets in $\gamma\gamma$ collisions



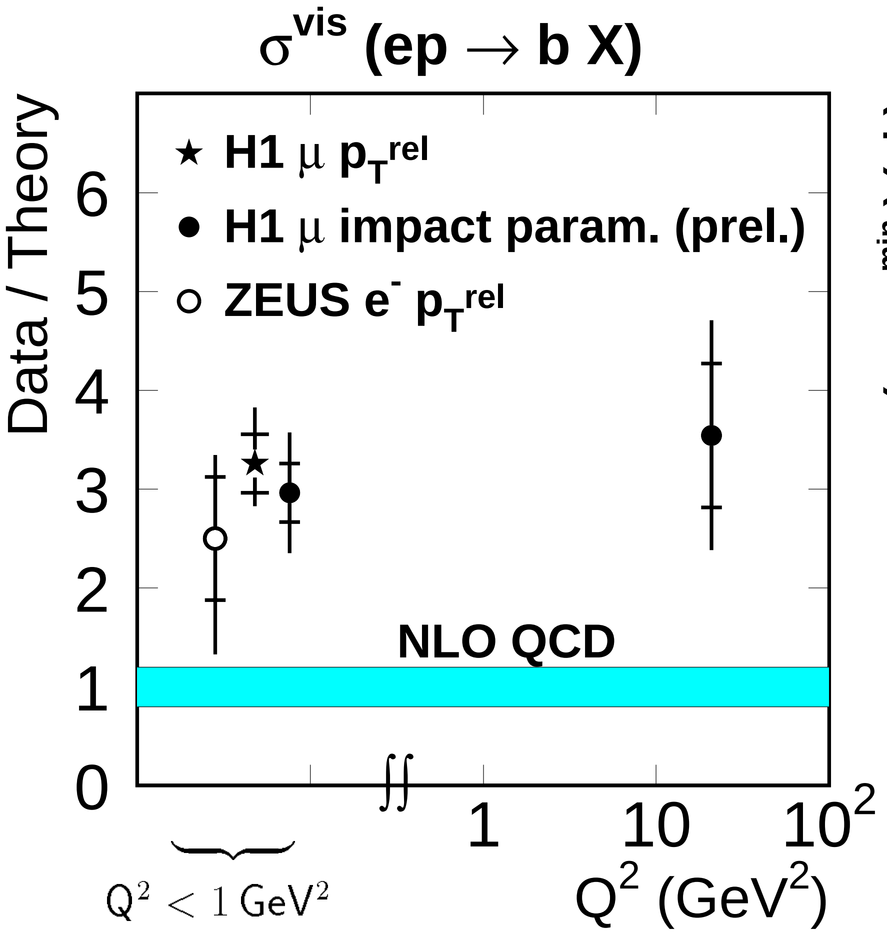
Good agreement of OPAL data **with NLO** calculations using GRV HO PDF in the region dominated by the **single resolved photon** contribution, where **soft underlying collision** (realized by multiparton interaction in PYTHIA) **does not occur.**

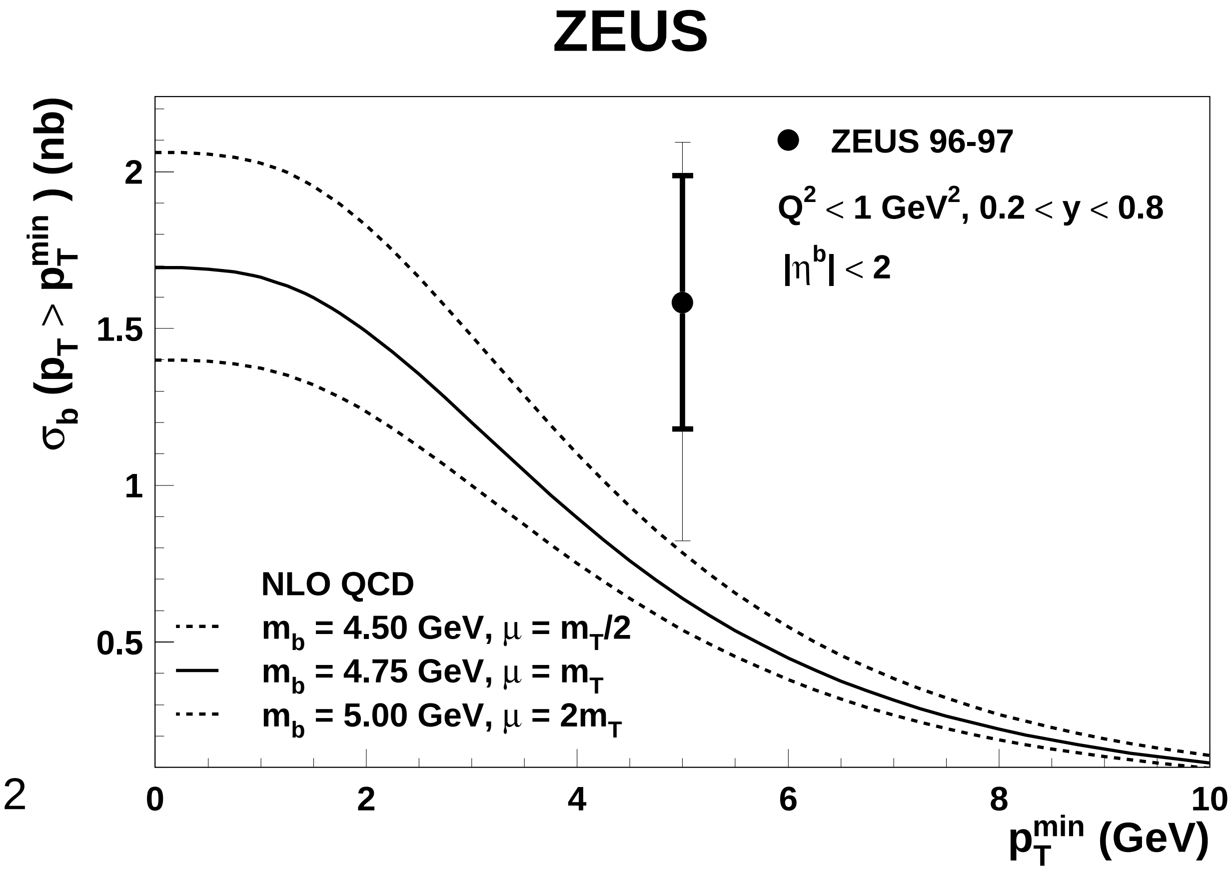


but significant **disagreement with NLO** calculations in the region dominated by the **double resolved photon** contribution, where multiparton interaction **is important.**

Implications for ep collisions at low E_T: **avoid processes influenced by soft underlying collisions!**

$\bar{b}b$ production in ep collisions



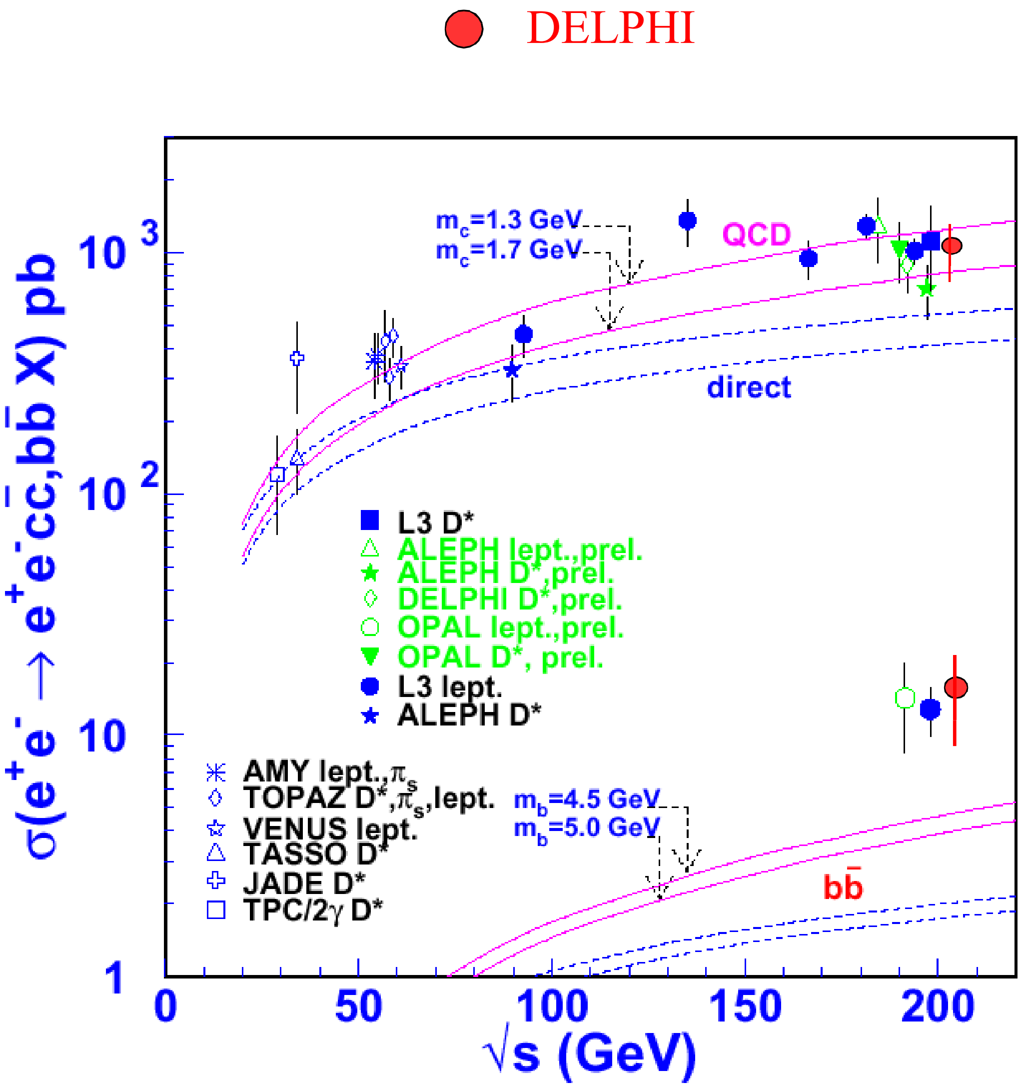


Clear **excess over PQCD** at $Q^2 = 0$ but data **not quite consistent** at high Q^2

$\bar{b}b$ production in $\gamma\gamma$ collisions

Comparison with L3 and OPAL

DELPHI



- New **DELPHI** data suggest **striking agreement** between the three LEP experiments
- and **dramatic disagreement** of their data with PQCD
- despite the fact that this process is expected to be the **cleanest test** of PQCD
- my view:
  current calculations are **not truly NLO QCD** because they **do not include** order $\alpha^2\alpha_s^2$ direct photon contribution.

QCD analysis of $\sigma(\gamma\gamma \to Q\overline{Q})$

In the conventional approach the **NLO QCD** approximation is defined by taking into account **the first two terms** in expansions of direct, as well as single and double resolved photon contributions

$$\begin{aligned}
\sigma_{\rm dir} &= \sigma^{(0)}_{\rm dir} + \sigma^{(1)}_{\rm dir}\alpha_s(\mu) + \sigma^{(2)}_{\rm dir}(M,\mu)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm dir}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm sr} &= \sigma^{(1)}_{\rm sr}(M)\alpha_s(\mu) + \sigma^{(2)}_{\rm sr}(M,\mu)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm sr}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm dr} &= \sigma^{(2)}_{\rm dr}(M)\alpha_s^2(\mu) + \sigma^{(3)}_{\rm dr}(M,\mu)\alpha_s^3(\mu) + \cdots
\end{aligned}$$

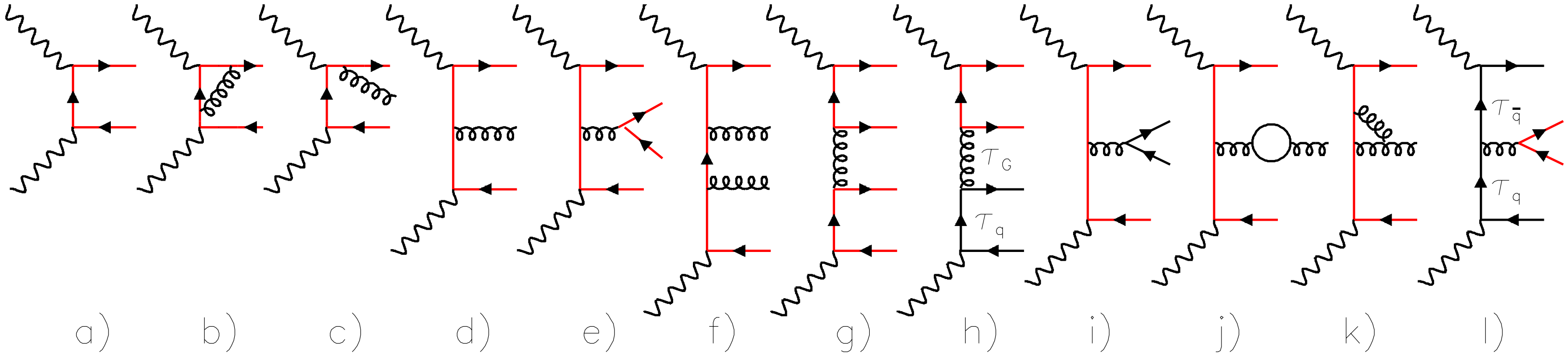
to the total cross section

$$\sigma(\gamma\gamma \to Q\overline{Q}) = \sigma_{\rm dir} + \sigma_{\rm sr} + \sigma_{\rm dr}.$$

where $\sigma^{(0)}_{\rm dir}$ comes from **pure QED** and equals

$$\sigma^{(0)}_{\rm dir} = \sigma_0 \left[\left(1 + \frac{4m_Q^2}{s} - \frac{8m_Q^4}{s^2}\right)\ln\frac{1+\beta}{1-\beta} - \beta\left(1 + \frac{4m_Q^2}{s}\right)\right],$$

Direct photon contribution to $\sigma(\gamma\gamma \to Q\overline{Q})$



At the order $\alpha^2\alpha_s^2$ diagrams with light quarks appear and we can distinguish (to all orders) **three classes** of contributions, differing by the charge factor CF :

Class A: $CF = e_Q^4$. Comes from diagrams in which both primary photons couple to heavy quarks or antiquarks.

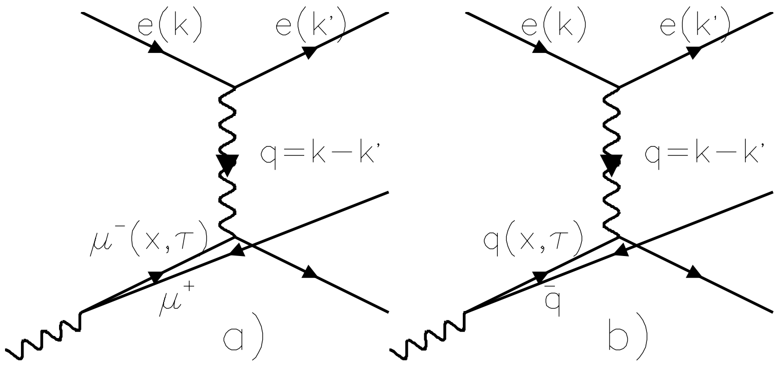
Class B: $CF = e_Q^2 e_q^2$. Comes from diagrams in which one of the primary photons couples to a heavy and the other to a light quark-antiquark pair.

Class C: $CF = e_q^4$. Comes from diagrams in which both photons couple to light $q\overline{q}$ pairs.

- For classes B and C the corresponding diagrams involve (and massless light quarks) **initial state mass singularities** that must be subtracted and put into the corresponding PDF of the photons.
- Because of different charge factors C_F classes A, B and C **do not mix** under **renormalization** of α_s and **factorization** of mass singularities.
- The order $\alpha^2\alpha_s^2$ direct photon contributions of **all three classes are needed** for theoretical consistency:
  - **A:** for the calculation of class A direct photon contribution (which does not mix with any other) to be performed in a **well-defined RS**
  - **B:** For **factorization scale invariance** of the sum of direct and resolved photon contributions.
  - **C:** dtto.
- Classes **B,C** can be defined also for the **PL parts of single** and class **C** for **PL parts of double** resolved photon contributions.
- Classes A,B,C can be treated **separately** as they **do not mix.**

Virtual photon structure: why

Operational definition of the **virtual photon**: $P^2 \gtrsim 1\ \mathrm{GeV}^2 \gg \Lambda^2_{\mathrm{QCD}}$. In this region the concept of **resolved virtual photon can be discarded** but there are good reasons for using it **provided the photon "lifetime" is much longer than the interaction time,** i.e. if $P^2 \ll M^2$.



cross section contains singular terms:

$$\int_{\tau_{min}}^{M^2} \mathrm{d}\tau \left[\frac{f(x)}{\tau} + \frac{g(x)m_q^2}{\tau^2} + \frac{h(x)P^2}{\tau^2}\right]$$

where

$$\tau^{\min} = xP^2 + \frac{m_q^2}{(1-x)}$$

After integration we get in units of $3e_q^2\alpha/2\pi$

$$q^{\mathrm{QED}}(x, m_q^2, P^2, M^2) = f(x)\ln\left(\frac{M^2}{\tau^{\min}}\right) + \left[-f(x) + \frac{g(x)m_q^2 + h(x)P^2}{\tau^{\min}}\right]\left(1 - \frac{\tau^{\min}}{M^2}\right)$$

$$f_T(x) = x^2 + (1-x)^2, \quad g_T(x) = \frac{1}{1-x}, \quad h_T(x) = 0,$$

$$f_L(x) = 0, \quad g_L(x) = 0, \quad h_L(x) = 4x^2(1-x).$$

This expression **vanishes for** $\tau^{\min} \to M^2$ and simplifies for $\tau^{\min} \ll M^2$

$$q^{\rm QED}(x, m_q^2, P^2, M^2) = f(x) \ln\left(\frac{M^2}{xP^2 + m_q^2/(1-x)}\right) - f(x) + \frac{g(x)m_q^2 + h(x)P^2}{xP^2 + m_q^2/(1-x)}.$$

$x(1-x)P^2 \ll m_q^2 \Rightarrow$: **real photon**, only γ_T,

$$q^{\rm QED}(x, m_q^2, 0, M^2) = (x^2 + (1-x)^2) \ln\left(\frac{M^2(1-x)}{m_q^2}\right) + 2x(1-x)$$

$x(1-x)P^2 \gg m_q^2 \Rightarrow$: both γ_T^* and γ_L^*

$$q_T^{\rm QED}(x, 0, P^2, M^2) = \left(x^2 + (1-x)^2\right) \ln\left(\frac{M^2}{xP^2}\right) + 2x(1-x) - 1$$

$$q_L^{\rm QED}(x, 0, P^2, M^2) = 4x(1-x)$$

The above terms come from the **collinear region** and are thus of **partonic** nature. There are **regular terms,** which also yield constant terms.

QCD: adds further **parton emissions** off the primary $\overline{q}q$ pair.

QCD-improved PDF of γ^*: a way how to include **part of higher order perturbative corrections that have clear partonic interpretation.**

Has the QCD renormalization-group-improved parton content of virtual photons been observed?

M. Glück, E. Reya, and I.Schienbein (PRD D63 (2001), 074008)

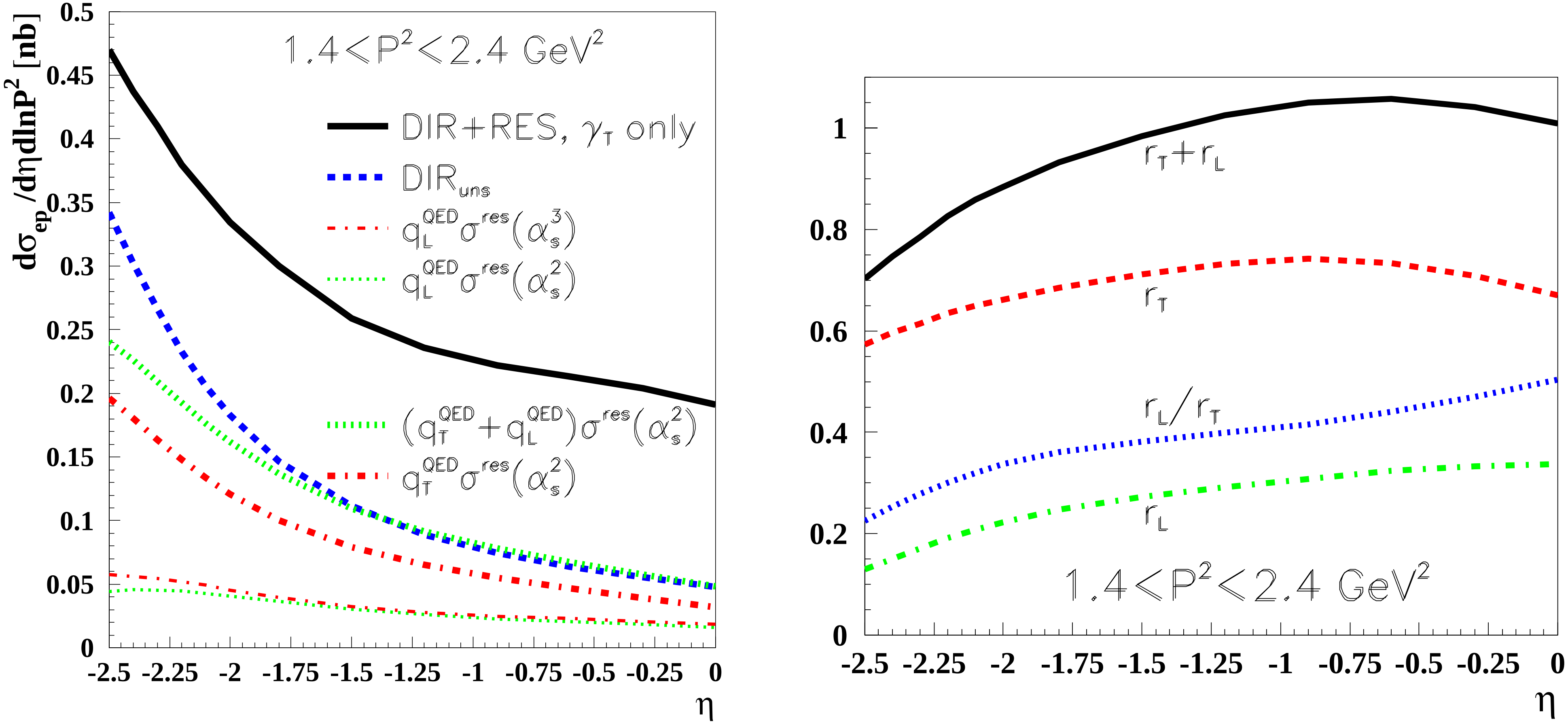
It is demonstrated that present e+e- and DIS ep data on the structure of the virtual photon can be understood entirely in terms of the standard naive quark-parton model box approach. Thus the QCD renormalization group (RG) improved parton distributions of virtual photons, in particular their gluonic component, have not yet been observed. The appropriate kinematical regions for their future observation are pointed out as well as suitable measurements which may demonstrate their relevance.

Still true? **Not quite!**

What to look for: deviation of data from the NLO direct photon QCD calculations as these include the QED part of resolved γ_T^* and γ_L^*.

H1 and ZEUS data on dijet production in low Q^2 region provide evidence for **effects beyond purely QED structure** of the photon.

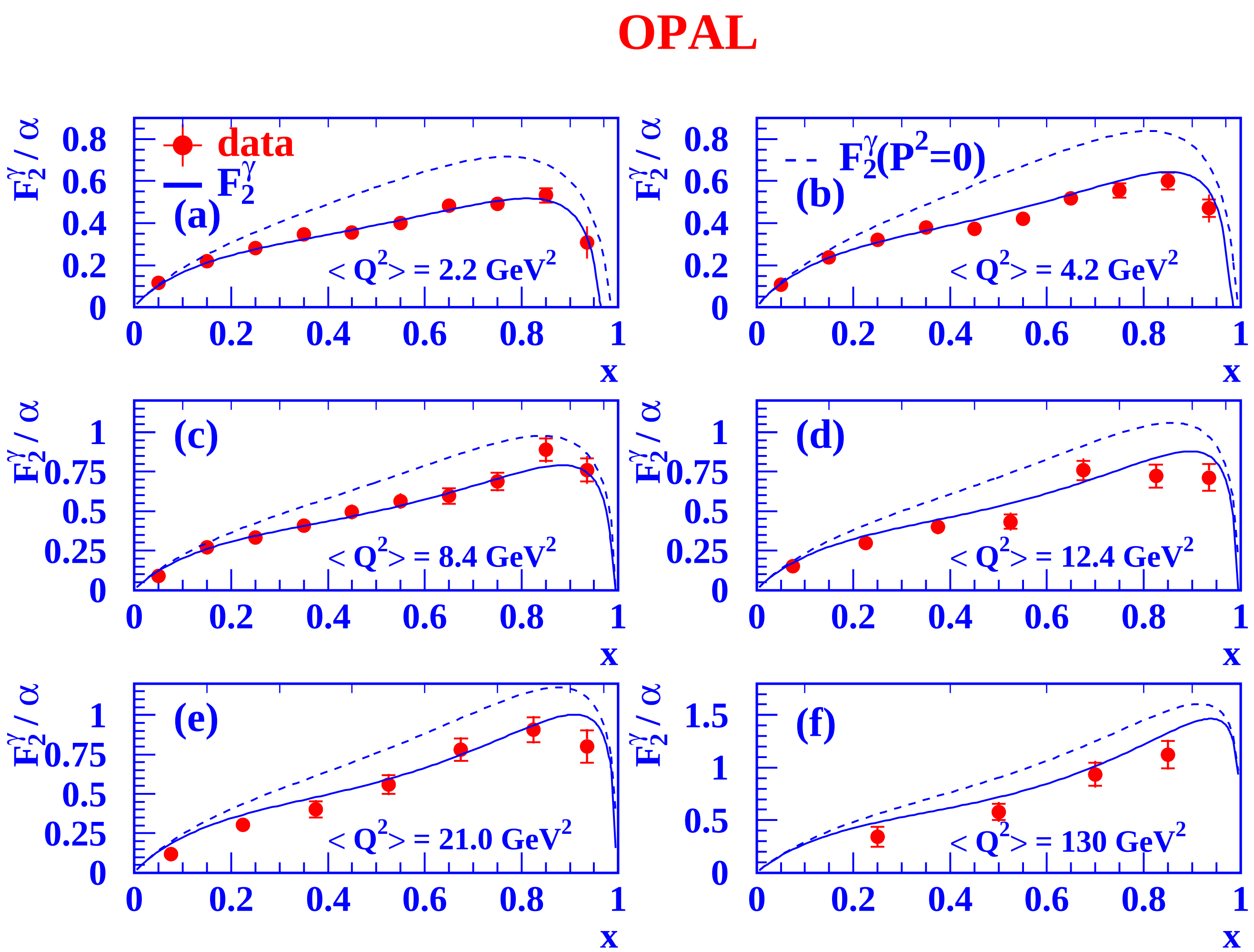
Order $\alpha\alpha_s^2$ and $\alpha\alpha_s^2$ JETVIP calculations



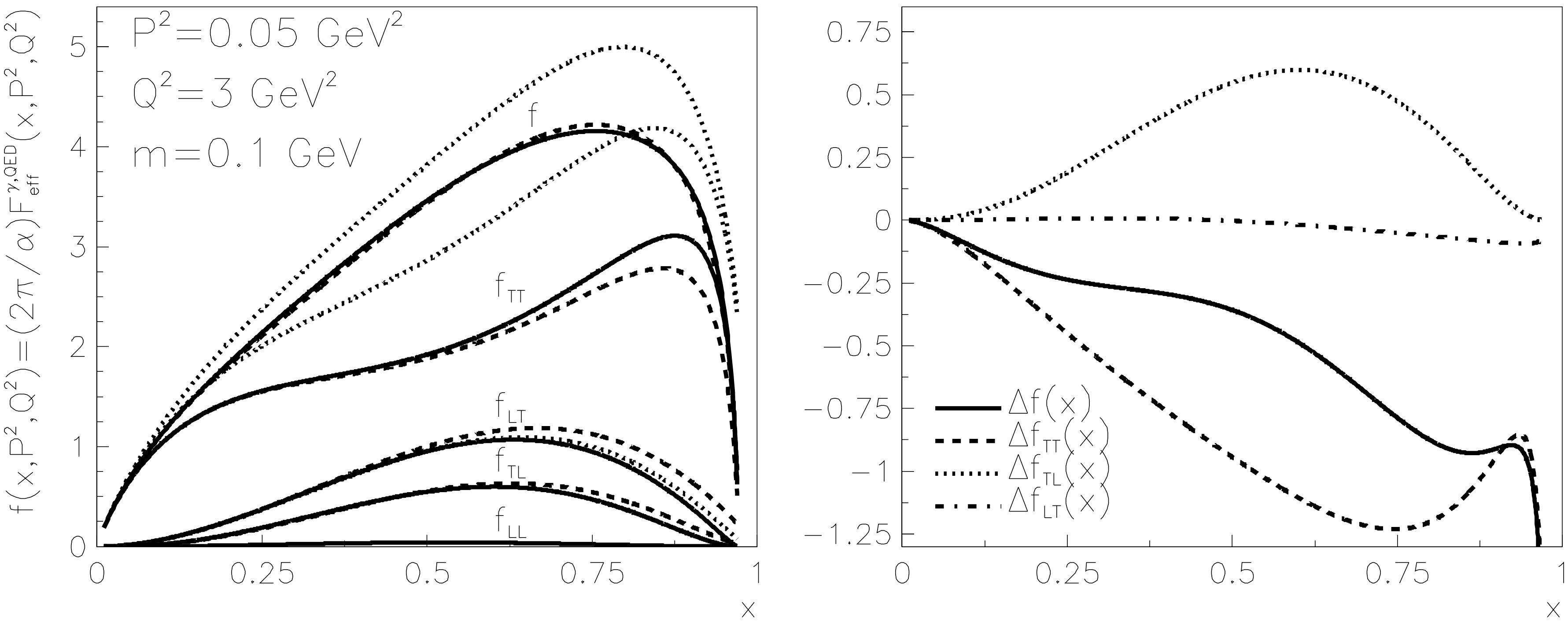
NLO QCD calculations **well approximated** by the sum of LO direct and LO resolved contributions evaluated with QED distributions of virtual photons.

Resolved longitudinal photons: why not?

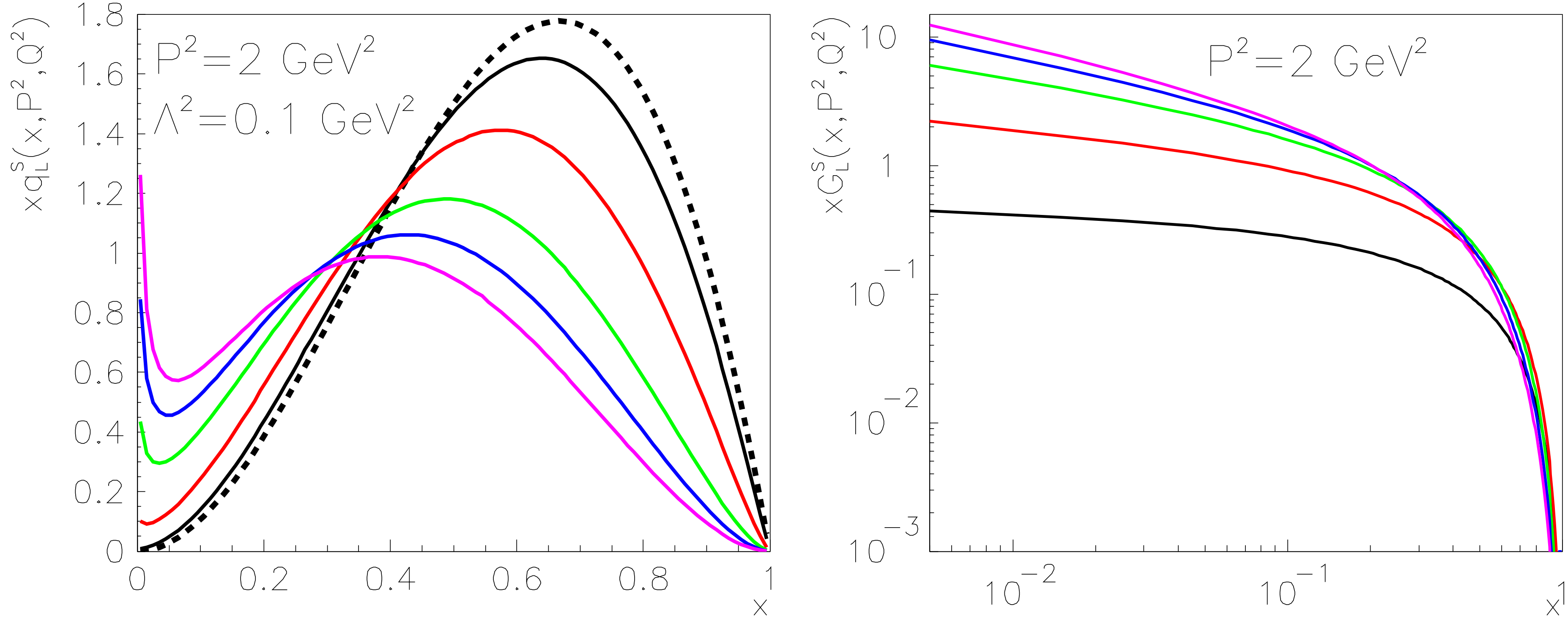
- Data on $F_2^{\gamma,\mathrm{QED}}(x,P^2,Q^2)$ for $Q^2 \gg P^2$ suggests it,
- its contribution has **clear partonic interpretation** and
- the **H1 data seem to need** it as well.
- But it is **not mandatory**.



QED contribution:



QCD evolution of γ_L^* similar to that of hadrons:



$M^2 = 3, 10, 100, 1000, 10000\ \mathrm{GeV}^2$ in decreasing order, dashed line corresponds to QED