Heavy quark production in $\gamma\gamma$ collisions

Jiří Chýla
Center for Particle Physics, Institute of Physics, Prague

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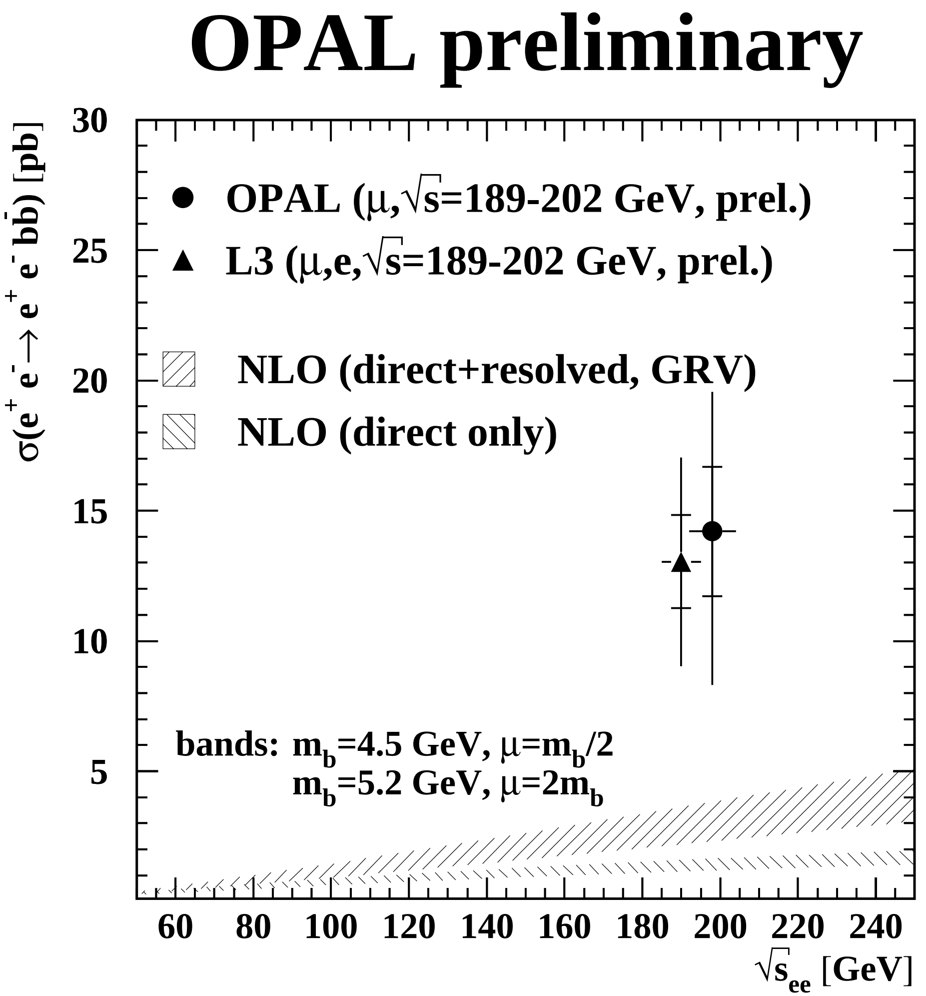
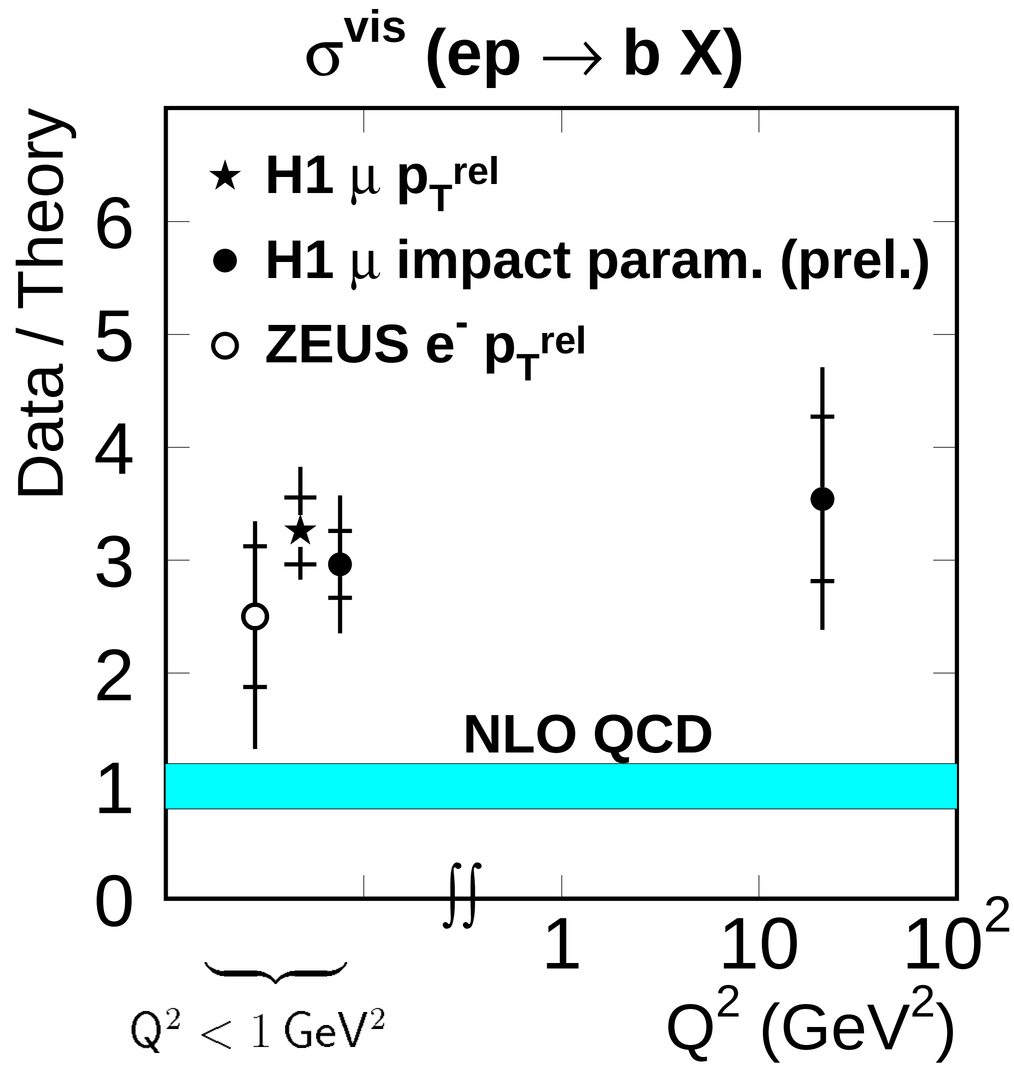
Details in **hep-ph/0010140**
Related in part to **JHEP04(2000)007**
Rejected by referees in **PLB** as well as **JHEP**

$b\bar{b}$ production in γp and $\gamma\gamma$ collisions: crisis of PQCD?





QCD analysis of $\sigma(\gamma\gamma \to Q\overline{Q})$: conventional approach

In the conventional approach the **NLO QCD** approximation is defined by taking into account **the first two terms** in expansions of direct, as well as single and double resolved photon contributions

$$\begin{aligned}
\sigma_{\rm dir} &= \sigma_{\rm dir}^{(0)} + \sigma_{\rm dir}^{(1)}\alpha_s(\mu) + \sigma_{\rm dir}^{(2)}(M,\mu)\alpha_s^2(\mu) + \sigma_{\rm dir}^{(3)}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm sr} &= \sigma_{\rm sr}^{(1)}(M)\alpha_s(\mu) + \sigma_{\rm sr}^{(2)}(M,\mu)\alpha_s^2(\mu) + \sigma_{\rm sr}^{(3)}(M,\mu)\alpha_s^3(\mu) + \cdots, \\
\sigma_{\rm dr} &= \sigma_{\rm dr}^{(2)}(M)\alpha_s^2(\mu) + \sigma_{\rm dr}^{(3)}(M,\mu)\alpha_s^3(\mu) + \cdots
\end{aligned}$$

to the total cross section

$$\sigma(\gamma\gamma \to Q\overline{Q}) = \sigma_{\rm dir} + \sigma_{\rm sr} + \sigma_{\rm dr}.$$

where $\sigma_{\rm dir}^{(0)}$ comes from **pure QED** and equals

$$\sigma_{\rm dir}^{(0)} = \sigma_0\left[\left(1+\frac{4m_Q^2}{s}-\frac{8m_Q^4}{s^2}\right)\ln\frac{1+\beta}{1-\beta} - \beta\left(1+\frac{4m_Q^2}{s}\right)\right],$$

PDF of the photon satisfy the system of **inhomogeneous** evolution equations

$$\begin{aligned}
\frac{\mathrm{d}\Sigma(x,M)}{\mathrm{d}\ln M^2} &= \delta_\Sigma k_q + P_{qq}\otimes\Sigma + P_{qG}\otimes G,\\
\frac{\mathrm{d}G(x,M)}{\mathrm{d}\ln M^2} &= k_G + P_{Gq}\otimes\Sigma + P_{GG}\otimes G,\\
\frac{\mathrm{d}q_{\mathrm{NS}}(x,M)}{\mathrm{d}\ln M^2} &= \delta_{\mathrm{NS}} k_q + P_{\mathrm{NS}}\otimes q_{\mathrm{NS}},
\end{aligned}$$

where $\delta_{\mathrm{NS}} \equiv 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right)$, $\delta_\Sigma = 6n_f\langle e^2\rangle$ and

$$\begin{aligned}
k_q(x,M) &= \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right],\\
k_G(x,M) &= \frac{\alpha}{2\pi}\left[\frac{\alpha_s(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right],\\
P_{ij}(x,M) &= \frac{\alpha_s(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots.
\end{aligned}$$

Due to the presence of k_q general solution of these equations can be split into the **hadron-like** (HAD) and **point-like** (PL) parts.

$$D(x,M) = D^{\mathrm{PL}}(x,M,M_0) + D^{\mathrm{HAD}}(x,M,M_0).$$

All complications of hard collisions of photons stem from $D^{\rm PL}(x, M, M_0)$ which results from the resummations

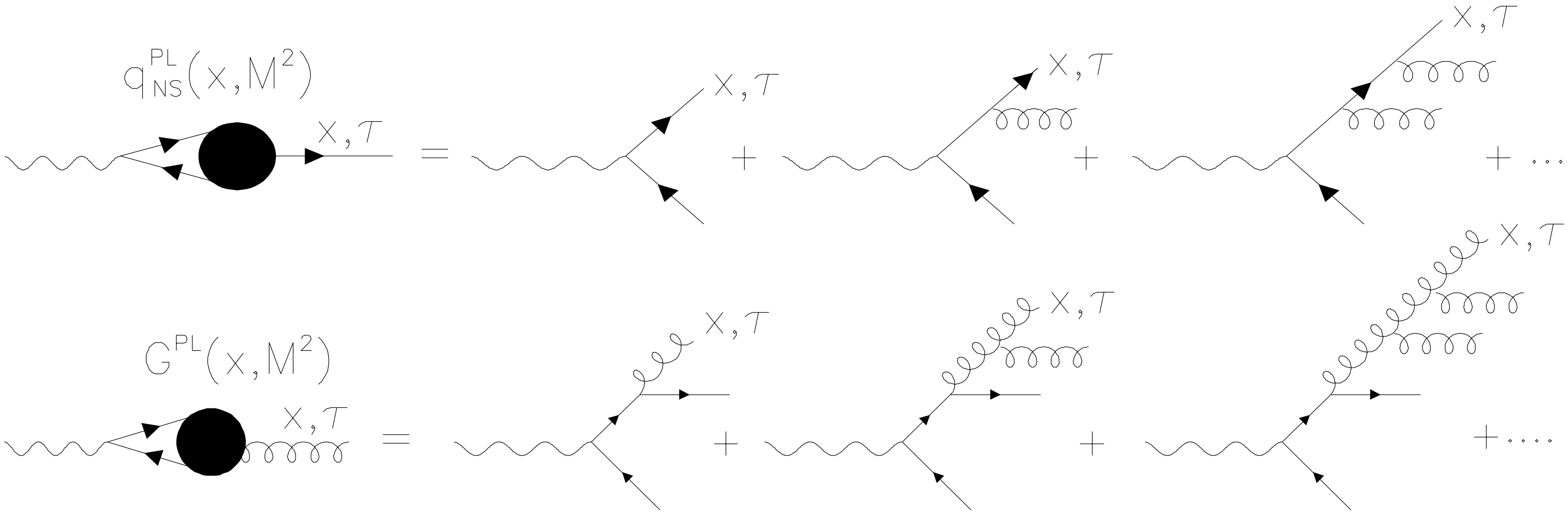


Figure 1: Diagrams defining the point-like parts of nonsinglet quark and gluon distribution functions. The resummation involves integration over parton virtualities $\tau \leq M^2$ and is represented by the junction of the blob and the $\gamma \to q\overline{q}$ vertex. Partons going into the hard collision are denoted by x, τ.

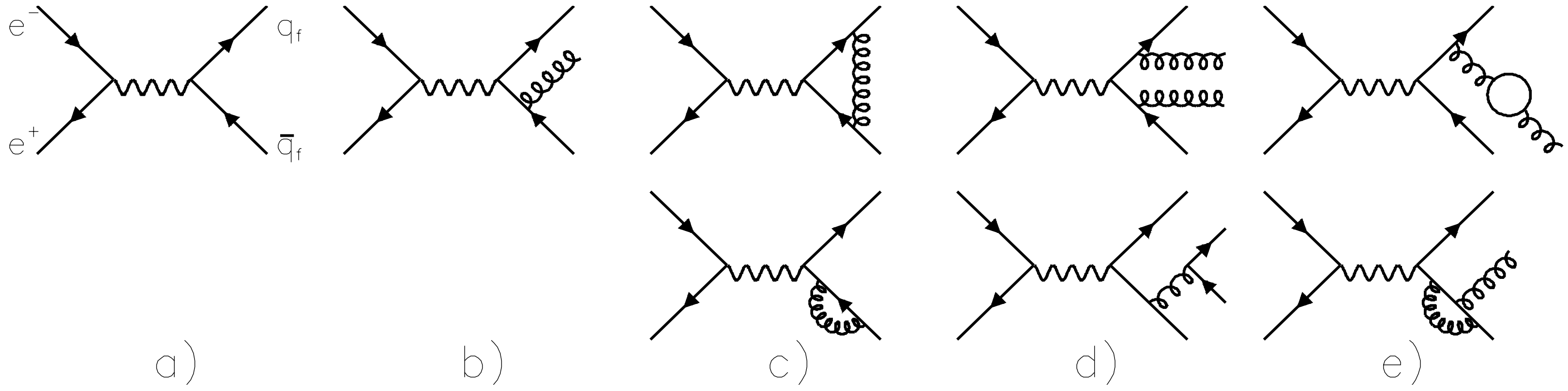
Defining the NLO approximation

Recall the definition of the term **next-to-leading** for $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons})$

$$\sigma_{\mathrm{had}}(Q) = \sigma^{(0)}_{\mathrm{had}}(Q) + \alpha_s(\mu)\sigma^{(1)}_{\mathrm{had}}(Q) + \alpha_s^2(\mu)\sigma^{(2)}_{\mathrm{had}}(Q/\mu) + \cdots = \sigma^{(0)}_{\mathrm{had}}(Q)(1 + r(Q)),$$

where $\sigma^{(0)}_{\mathrm{had}}(Q) \equiv (4\pi\alpha^2/Q^2)\sum_f e_f^2$ comes from **pure QED** whereas **genuine QCD effects** are contained in

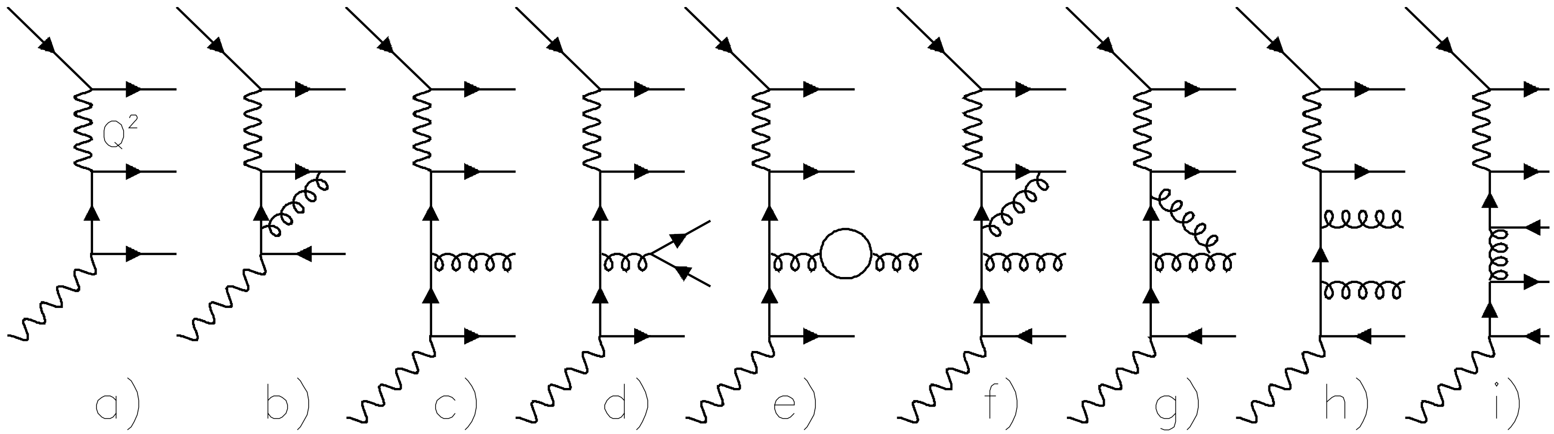
$$r(Q) = \frac{\alpha_s(\mu)}{\pi}\left[1 + \frac{\alpha_s(\mu)}{\pi} r_1(Q/\mu) + \cdots\right]. \tag{1}$$



For this quantity the terms **LO** and **NLO** are applied to **genuine QCD effects!**

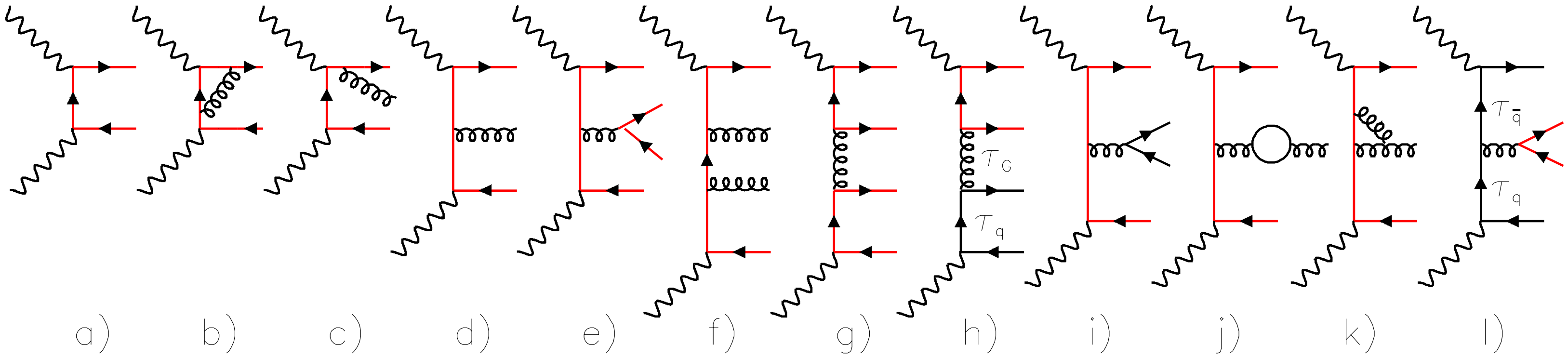
NOTE: inclusion of first two term in (**??**) is needed in order to work in a **well-defined renormalization scheme of α_s!**

For purely perturbative quantities like $\sigma_{\rm had}(Q)$ the association of the term **NLO QCD approximation** with a **well-defined RS** is a **generally accepted convention**, worth retaining for physical quantities in any hard scattering process, like the point-like part of $F_2^\gamma(x,Q^2)$, measured in DIS on the photon:



and discussed in detail in **JHEP04(2000)007**.

Direct photon contribution to $\sigma(\gamma\gamma \to Q\overline{Q})$



At the order $\alpha^2\alpha_s^2$ diagrams with light quarks appear and we can distinguish (to all orders) **three classes** of contributions, differing by the charge factor CF :

Class A: $CF = e_Q^4$. Comes from diagrams in which both primary photons couple to heavy quarks or antiquarks.

Class B: $CF = e_Q^2 e_q^2$. Comes from diagrams in which one of the primary photons couples to a heavy and the other to a light quark-antiquark pair.

Class C: $CF = e_q^4$. Comes from diagrams in which both photons couple to light $q\overline{q}$ pairs.

- For classes B and C the corresponding diagrams involve (and massless light quarks) **initial state mass singularities** that must be subtracted and put into the corresponding PDF of the photons.
- Because of different charge factors C_F classes A, B and C **do not mix** under **renormalization** of α_s and **factorization** of mass singularities.
- The order $\alpha^2\alpha_s^2$ direct photon contributions of **all three classes are needed** for theoretical consistency:
  - **A:** for the calculation of class A direct photon contribution (which does not mix with any other) to be performed in a **well-defined RS**
  - **B:** For **factorization scale invariance** of the sum of direct and resolved photon contributions.
  - **C:** dtto.
- Classes **B,C** can be defined also for the **PL parts of single** and class **C** for **PL parts of double** resolved photon contributions.
- Classes A,B,C can be treated **separately** as they **do not mix.**

Class A direct photon contributions

In the conventional approach the "leading" and "nexto-to-leading" order approximations of $\sigma_{\rm dir}$ are defined as follows

LO: $\sigma^{(0)}_{\rm dir}$, which is of **purely QED** origin

NLO: $\sigma^{(01)}_{\rm dir} \equiv \sigma^{(0)}_{\rm dir} + \alpha_s(\mu)\sigma^{(1)}_{\rm dir}$

But the latter expression has the **same form** as the first two terms in $\sigma_{\rm had}(Q)$

$\Downarrow$

$\sigma^{(01)}_{\rm dir}$ cannot be associated to a well-defined RS

$\Downarrow$

does not deserve the label NLO!

For QCD analysis of $\sigma_{\rm dir}$ in a well-defined RS the the **third term** in $\sigma_{\rm dir}$, proportional to $\alpha_s^2(\mu)$ is **indispensable**. We need in particular the **diagrams involving loops** which contribute to the renormalization of $\alpha_s(\mu)$. The regular, μ-dependent parts of their contributions provide the $\ln\mu^2$ term canceling the μ dependence of $\alpha_s(\mu)$ in the second term of $\sigma^{(01)}_{\rm dir}$.

The lesson from $Q\overline{Q}$ production in pp collisions

Recall how factorization operates for heavy quark production in $\overline{\rm p}$p collisions, where at the NLO

$$\sigma^{\rm NLO}(\overline{\rm p}{\rm p}\rightarrow Q\overline{Q})=D_1(M)\otimes\left[\alpha_s^2(\mu)\sigma^{(2)}+\alpha_s^3(\mu)\sigma^{(3)}(M,\mu)\right]\otimes D_2(M),$$

and $D_1(M)$ and $D_2(M)$ satisfy the **homogeneous** evolution equations. Factorization scale invariance of this expression requires that

$$\frac{{\rm d}\sigma^{\rm NLO}(\overline{\rm p}{\rm p}\rightarrow Q\overline{Q})}{{\rm d}\ln {\rm M}}=\alpha_s^2(\mu)\left[\frac{{\rm d}D_1(M)}{{\rm d}\ln M}\otimes\sigma^{(2)}\otimes D_2(M)+D_1(M)\otimes\sigma^{(2)}\otimes\frac{{\rm d}D_2(M)}{{\rm d}\ln M}+\right.$$

$$\left.\alpha_s(\mu)D_1(M)\otimes\frac{{\rm d}\sigma^{(3)}(M)}{{\rm d}\ln M}\otimes D_2(M)\right]+$$

$$\alpha_s^3(\mu)\left(\frac{{\rm d}D_1(M)}{{\rm d}\ln M}\otimes\sigma^{(3)}\otimes D_2(M)+D_1(M)\otimes\sigma^{(3)}\otimes\frac{{\rm d}D_2(M)}{{\rm d}\ln M}\right)\propto\alpha_s^4 D_1(M)\otimes D_2(M)$$

which **is guaranteed** by the fact that

$$\left[\,.....\right]\propto\alpha_s^2,\qquad\left(\,.....\right)\propto\alpha_s\quad\text{due to}\quad\frac{{\rm d}D_i(M)}{{\rm d}\ln M}\propto\alpha_s(M)D_i(M)$$

Resolved photon contribution to $\sigma(\gamma\gamma \to Q\overline{Q})$

There are **5 classes** of resolved photon contributions:

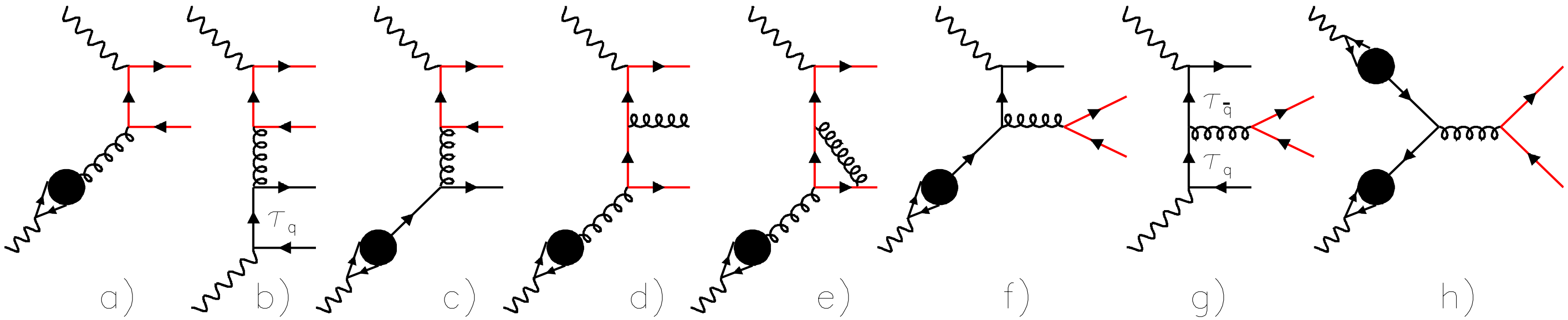
- **Single resolved** photon using
  - hadron-like parts of PDF ($\sigma_{\rm srh}$),
  - point-like parts of PDF ($\sigma_{\rm srp}$).
- **Double resolved** photon using
  - hadron-like parts of PDF on both sides ($\sigma_{\rm drhh}$),
  - hadron-like parts of PDF on one side and point-like on the other ($\sigma_{\rm drhp}$),
  - point-like parts of PDF on both sides ($\sigma_{\rm drpp}$).

With this subdivision of $\sigma_{\rm sr}$ and $\sigma_{\rm dr}$ in mind we can write

$$\sigma(\gamma\gamma \to Q\overline{Q}) = \sigma_{\rm dir} + \sigma_{\rm srh} + \sigma_{\rm srp} + \sigma_{\rm drhh} + \sigma_{\rm drhp} + \sigma_{\rm drpp}.$$

Only cross sections involving **point-like parts** of PDF will be considered further

$Q\overline{Q}$ production in single resolved photon contribution



$$\sigma^{(12)}_{\rm srp} \equiv \alpha_s(\mu)\sigma^{(1)}_{\rm srp}(M)+\alpha_s^2(\mu)\sigma^{(2)}_{\rm srp}(M,\mu) = \alpha_s(\mu)\left[\sigma^{(1)}\otimes G(M)+\alpha_s(\mu)\sigma^{(2)}(M,\mu)\otimes D(M)\right]$$

Class B single resolved photon terms **mix with direct** photon ones

$$\frac{{\rm d}\sigma^{(12)}_{\rm srp}}{{\rm d}\ln M} = \alpha_s(\mu)\left[\sigma^{(1)}\otimes\frac{{\rm d}G(M)}{{\rm d}\ln M}+\alpha_s(\mu)\frac{{\rm d}\sigma^{(2)}(M)}{{\rm d}\ln M}\otimes D(M)+\alpha_s(\mu)\sigma^{(2)}(M)\frac{{\rm d}D(M)}{{\rm d}\ln M}\right]$$

$$\frac{{\rm d}G(M)}{{\rm d}\ln M}\propto\alpha\alpha_s,\quad \frac{{\rm d}q(M)}{{\rm d}\ln M}\propto\alpha,\ \textbf{but}\ \left[...\right]\propto\alpha\alpha_s$$

only by **including class B direct photon contribution** which have the form

$$\alpha_s^2(\mu)\sigma^{(2)}_{\rm dir}(B)\propto\alpha\alpha_s^2(\mu)\ln M\Rightarrow\frac{{\rm d}\alpha_s^2(\mu)\sigma^{(2)}_{\rm dir}(B)}{{\rm d}\ln M}\propto\alpha\alpha_s^2$$

can the factorization scale invariance be guaranteed!

Phenomenological consequences

Conventional expression for the NLO approximation

$$\sigma^{\rm conv} \equiv \sigma_{\rm dir}^{(01)}(\mu) + \left[\sigma_{\rm sr}^{(12)}(M,\mu) + \sigma_{\rm dr}^{(23)}(M,\mu)\right]$$

where first term **does not mix** with the sum in [..], **is incomplete** because

- the **direct photon** contribution $\sigma_{\rm dir}^{(01)}$ is of the **LO only**,
- the **resolved photon** contribution is **not factorization scale invariant**.

Both shortcomings stem from **the absence of direct photon contributions of the order $\alpha^2\alpha_s^2$**. These come in **three classes**, each of them needed for different reasons to make the expression

$$\sigma^{\rm NLO} \equiv \sigma_{\rm dir}^{(02)}(A; M,\mu) + \left[\sigma_{\rm sr}^{(12)}(M,\mu) + \sigma_{\rm dr}^{(23)}(M,\mu) + \sigma_{\rm dir}^{(02)}(B, M,\mu) + \sigma_{\rm dir}^{(02)}(C, M,\mu)\right]$$

genuine NLO QCD approximation. What to do when $\sigma_{\rm dir}^{(2)}$ is not available? Check the **stability** of $\sigma^{\rm conv}$ by

investigating its factorization scale dependence!