QCD analysis of $F_2^\gamma(x,Q^2)$: an unconventional view

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- **Motivation**
- **Conventional formulation**
- **Alternative:**
  - **Separating QED effects**
  - F_2^γ **and evolution equations**
  - **Numerical results**
- **Conclusions**

Details in
J. Ch.,: JHEP04(2000)007, hep-ph/9911413

Motivation

Twofold:

- Clarify the meaning of the concepts **"LO"** and **"NLO"** in photon induced hard processes.
- Disentangle **genuine QCD** effects from those of **pure QED**.

My proposal builds in part on arguments advocated for a long time by **J. Field** and **F. Kapusta** and agrees with the approach to calculations of direct photon production at HERA pursued by **M. Krawczyk**.

Notation and basic formulae

$$\begin{aligned}\frac{1}{x}F_2^\gamma(x,Q^2) &= q_{\rm NS}(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}\delta_{\rm NS}C_\gamma + \\ &\langle e^2\rangle\Sigma(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}\langle e^2\rangle\delta_\Sigma C_\gamma + \\ &\langle e^2\rangle G(M)\otimes C_G(Q/M)\end{aligned}$$

where PDF of the photon satisfy the evolution equations

$$\begin{aligned}\frac{{\rm d}\Sigma(x,M)}{{\rm d}\ln M^2} &= \delta_\Sigma k_q + P_{qq}\otimes\Sigma + P_{qG}\otimes G,\\ \frac{{\rm d}G(x,M)}{{\rm d}\ln M^2} &= k_G + P_{Gq}\otimes\Sigma + P_{GG}\otimes G,\\ \frac{{\rm d}q_{\rm NS}(x,M)}{{\rm d}\ln M^2} &= \delta_{\rm NS}k_q + P_{\rm NS}\otimes q_{\rm NS},\end{aligned}$$

with quark nonsinglet and singlets defined as

$$\begin{aligned}\Sigma(x,M) &\equiv \sum_{i=1}^{n_f} q_i^+(x,M) \equiv \sum_{i=1}^{n_f}\left[q_i(x,M)+\overline{q}_i(x,M)\right],\\ q_{\rm NS}(x,M) &\equiv \sum_{i=1}^{n_f}\left(e_i^2-\langle e^2\rangle\right)\left(q_i(x,M)+\overline{q}_i(x,M)\right),\end{aligned}$$

$$\delta_{\rm NS} = 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right), \quad \delta_\Sigma = 6n_f\langle e^2\rangle.$$

PDF separated into **hadronic** and **pointlike** parts

$$D(x,M) = D^{\rm PL}(x,M) + D^{\rm HAD}(x,M).$$

both of which contain **QCD effects.**

$$k_q = \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right],$$

$$k_G = \frac{\alpha}{2\pi}\left[\qquad \frac{\alpha_s(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right],$$

$$P_{ij} = \frac{\alpha_s(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots,$$

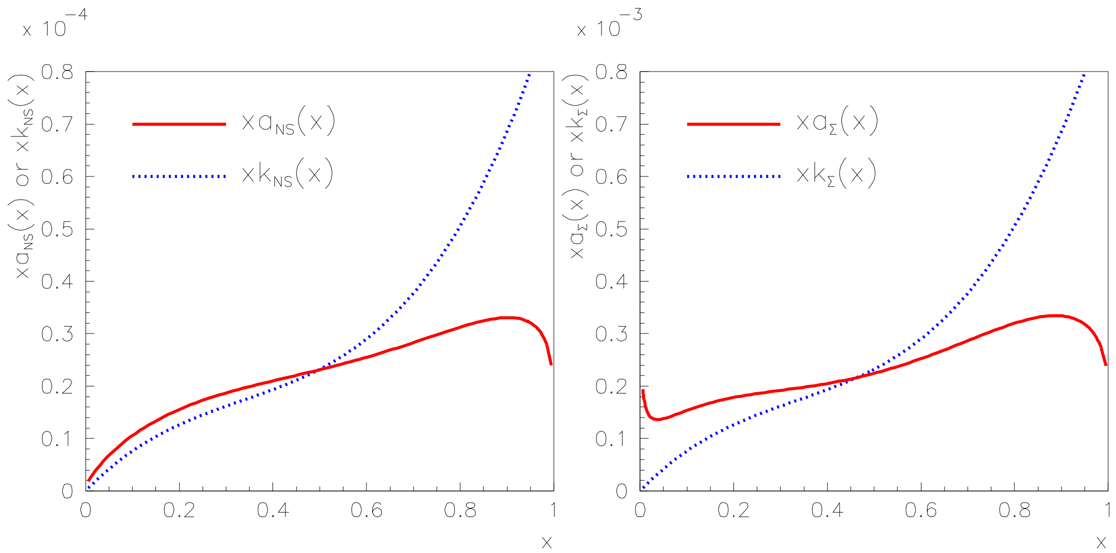
where $k_q^{(0)}(x) = (x^2 + (1-x)^2)$ and

$$C_q(x,Q/M) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(x,Q/M) + \cdots,$$

$$C_G(x,Q/M) = \frac{\alpha_s(\mu)}{2\pi}C_G^{(1)}(x,Q/M) + \cdots,$$

$$C_\gamma(x,Q/M) = C_\gamma^{(0)}(x,Q/M) + \frac{\alpha_s(\mu)}{2\pi}C_\gamma^{(1)}(x,Q/M) + \cdots,$$

$$C_\gamma^{(0)}(x,Q/M) = \left(x^2 + (1-x)^2\right)\left[\ln\frac{M^2}{Q^2} + \ln\frac{1-x}{x}\right] + 8x(1-x) - 1$$



Basic question: where to truncate these expansions?

Conventional formulation: nonsinglet channel at the LO

The light quark contribution to the PL part of $F_{\rm NS}^{\gamma}$

$$\frac{1}{x}F_{\rm NS}^{\gamma}(x,Q^2) = \delta_{\rm NS}\left[q(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}C_\gamma(Q/M)\right] =$$
$$\delta_{\rm NS}\left[q(M)+\frac{\alpha_s}{2\pi}q(M)\otimes C_q^{(1)}(Q/M)+\frac{\alpha}{2\pi}C_\gamma^{(0)}(Q/M)\right.$$
$$\left.+\frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}C_\gamma^{(1)}(Q/M)\cdots\right]$$

where $q \equiv u/3e_u^2 = d/3e_d^2 = s/3e_s^2$.

The conventional approach is based on two assumptions

- $F_{\rm NS}^{\gamma}$ expressed (dropping $\delta_{\rm NS}$) in terms of q as $F_{\rm NS}^{\rm p}$:

$$F_{\rm NS,LO}^{\gamma}(x,Q^2) = q_{\rm LO}(x,M)$$

- $q_{\rm LO}$ satisfies the evolution equation with r.h.s. including $k_q^{(0)}$ and $P_{qq}^{(0)}$ only.

Note: the **pure QED** quantity $C_\gamma^{(0)}$ is assigned to NLO! Consistency with evolution eqs. and **factorization scale independence** of $F_{\rm NS}^{\gamma}$ requires that

$$q(x,M^2) = \mathcal{O}(\alpha/\alpha_s)$$

because only then

$$\alpha_s(M)\left(q\otimes C_q^{(1)}\right) \approx \alpha C_\gamma^{(0)} = \mathcal{O}(\alpha)$$

is of the *"next-to-leading"* order with respect to q!

Seemingly this is also suggested by the explicit form of the PL solutions:

$$q_{\rm NS}^{\rm PL}(n,M_0,M)=\frac{4\pi}{\alpha_s(M)}\left[1-\left(\frac{\alpha_s(M)}{\alpha_s(M_0)}\right)^{1-2P_{qq}^{(0)}(n)/\beta_0}\right]a_{\rm NS}(n)$$

where

$$a_{\rm NS}(n)\equiv\frac{\alpha}{2\pi\beta_0}\frac{k_{\rm NS}^{(0)}(n)}{1-2P_{qq}^{(0)}(n)/\beta_0}$$

All PL solutions share **the same large M behavior**

$$q_{\rm NS}^{\rm PL}(x,M_0,M)\rightarrow\frac{4\pi}{\alpha_s(M)}a_{\rm NS}(x)\equiv q_{\rm NS}^{\rm AP}(x,M)\propto\ln\frac{M^2}{\Lambda^2}$$

defining the **asymptotic pointlike** solution $q_{\rm NS}^{\rm AP}$.

BUT: the fact that $\alpha_s(M)$ appears in the denominator of $q_{\rm NS}^{\rm AP}$ **cannot** be interpreted as evidence that

$$q(x,M)=\mathcal{O}(\alpha/\alpha_s)$$

because provided M_0 is kept fixed when $\alpha_s\rightarrow 0$

$$q_{\rm NS}^{\rm PL}(x,M,M_0)\rightarrow\frac{\alpha}{2\pi}k_{\rm NS}^{(0)}(x)\ln\frac{M^2}{M_0^2}$$

corresponding to **purely QED** splitting $\gamma\rightarrow q\overline{q}$.

Alternative formulation – the NS channel

Based on two related ingredients:

- **Separation of purely QED** effects, which actually **dominate** scaling violations of $F_{\rm NS}^{\gamma}(x,Q^2)$, in particular its $\ln Q^2$ rise, from **genuine QCD** ones. To identify the latter one has to look for subtler effects, like the **x-dependence** of the slope

$$a(x) \equiv \frac{\mathrm{d}F_{\rm NS}^{\gamma}(x,Q^2)}{\mathrm{d}\ln Q^2}$$

  or **low x** behaviour of $F_2^{\gamma}(x,Q^2)$.

- **Proper treatment of α_s dependence of PDF** in perturbation theory, i.e. as $\alpha_s \to 0$:

$$q(x,M), G(x,M) \propto (\alpha \ln M^2) = \mathcal{O}(\alpha)$$

  rather than

$$q(x,M), G(x,M) \propto (\alpha/\alpha_s) = \mathcal{O}(\alpha/\alpha_s)$$

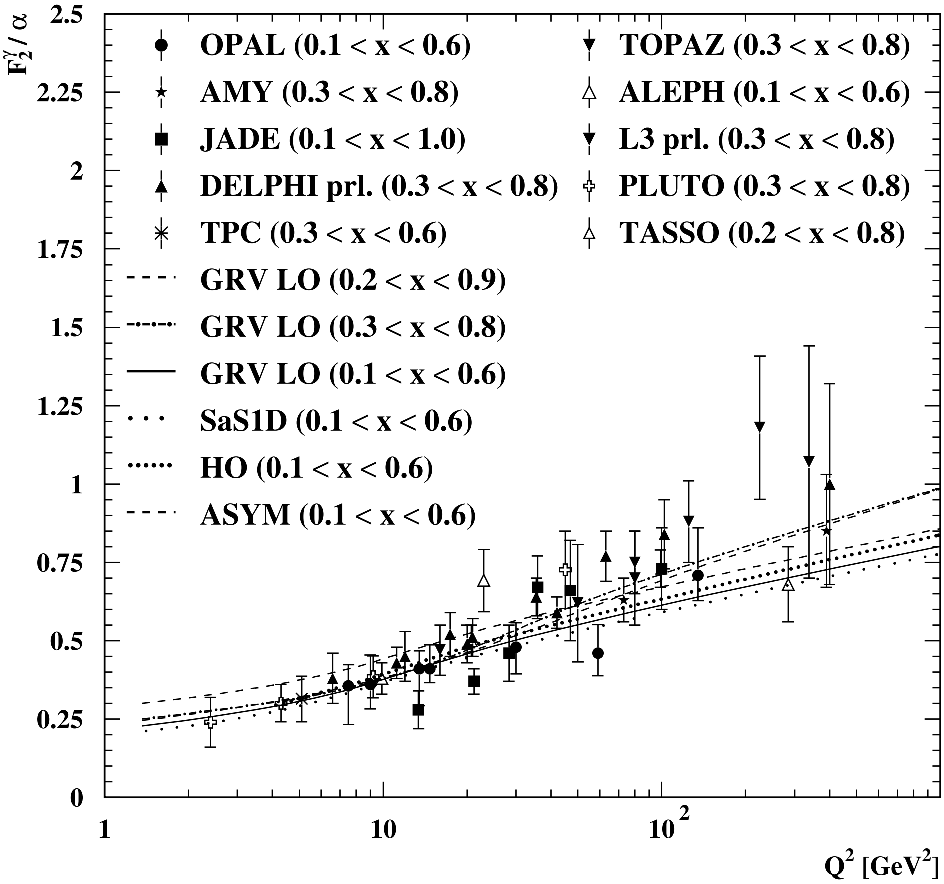
  as in the conventional approach (recall my dispute with A. Vogt at PHOTON'99)

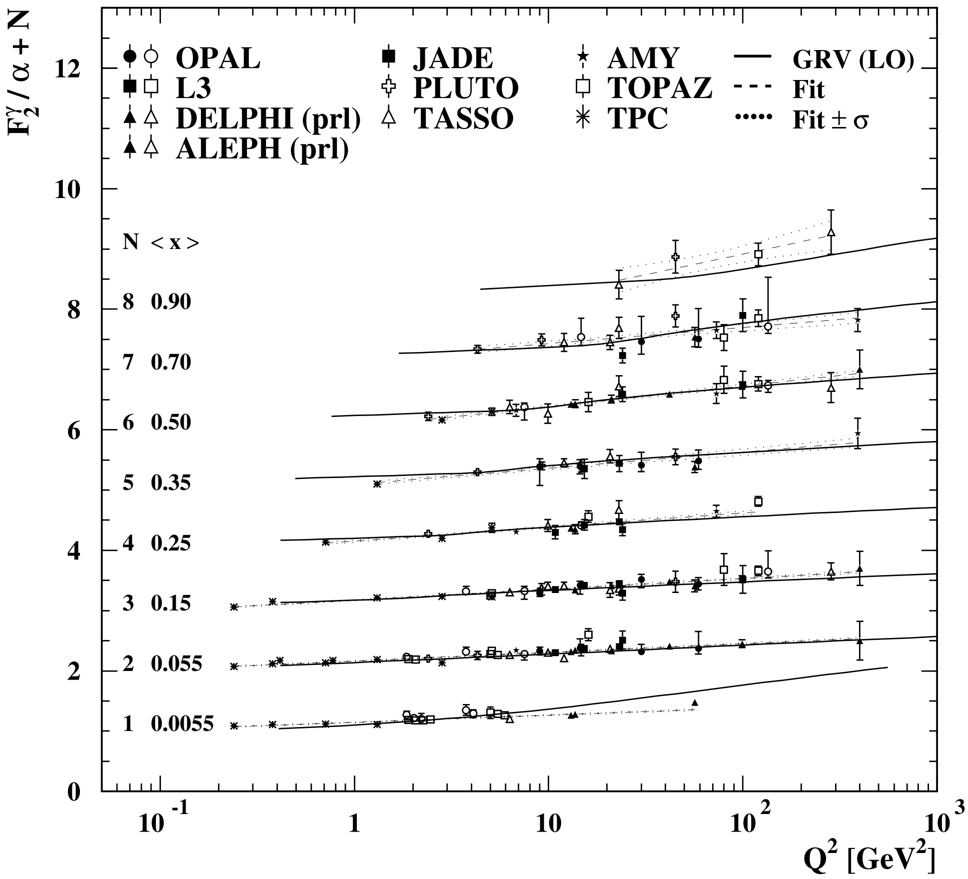
  The point is simple:

$$\ln M^2 \neq \frac{1}{\alpha_s}$$

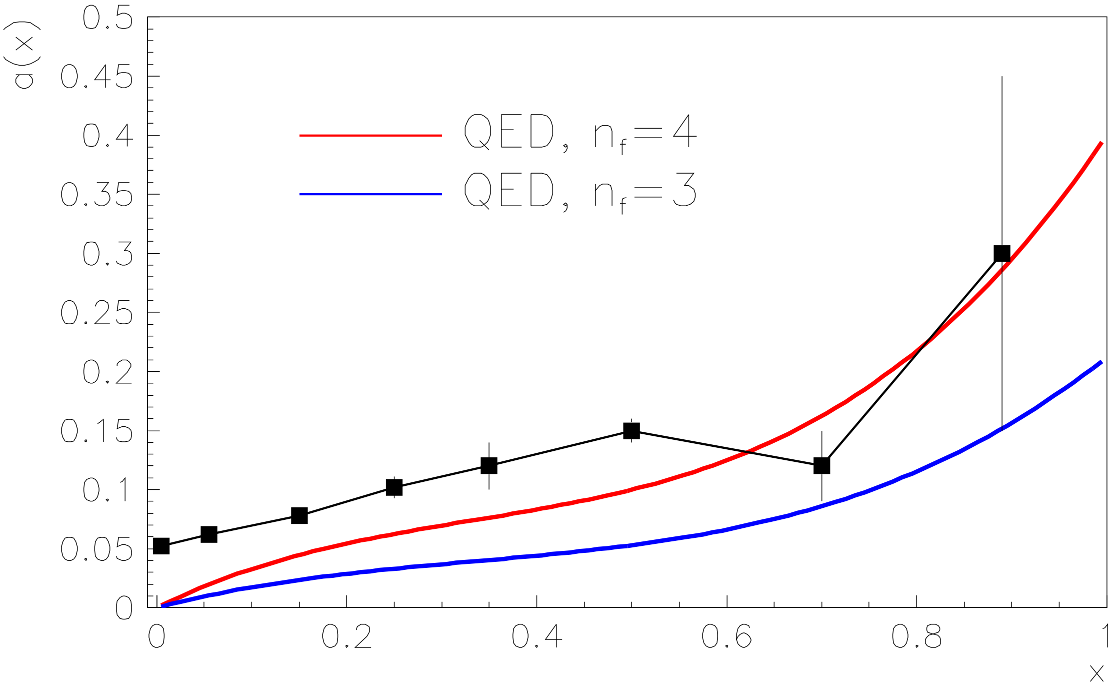
  as the **log comes from pure QED!**

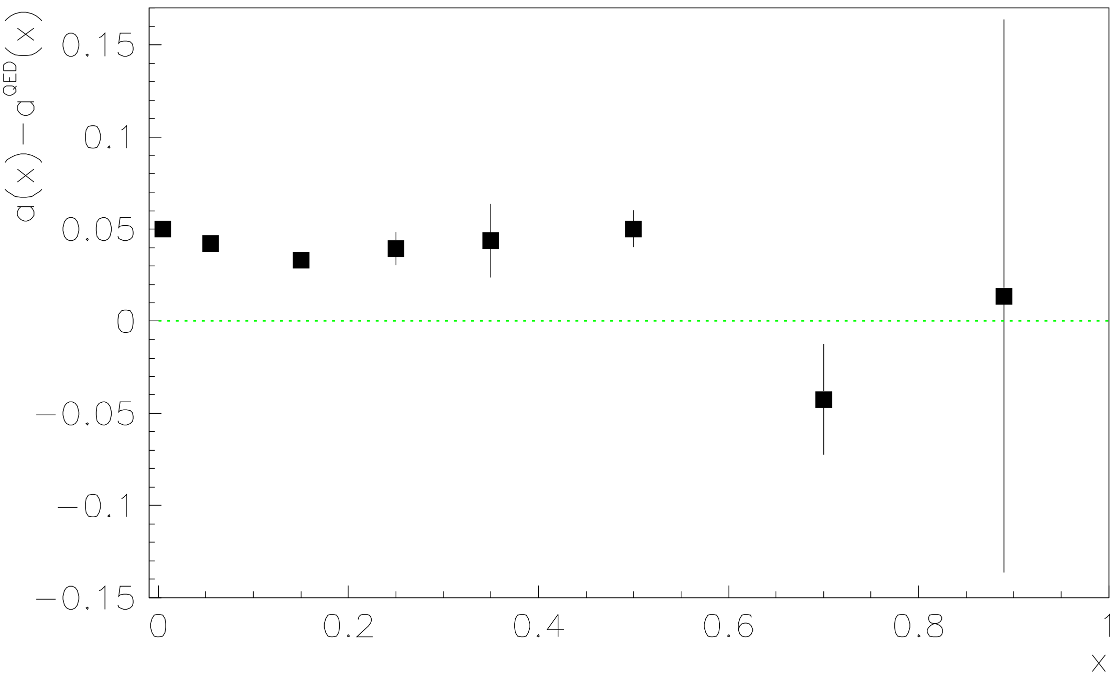
Vast difference in precision and scope of data on F_2^γ and F_2^p

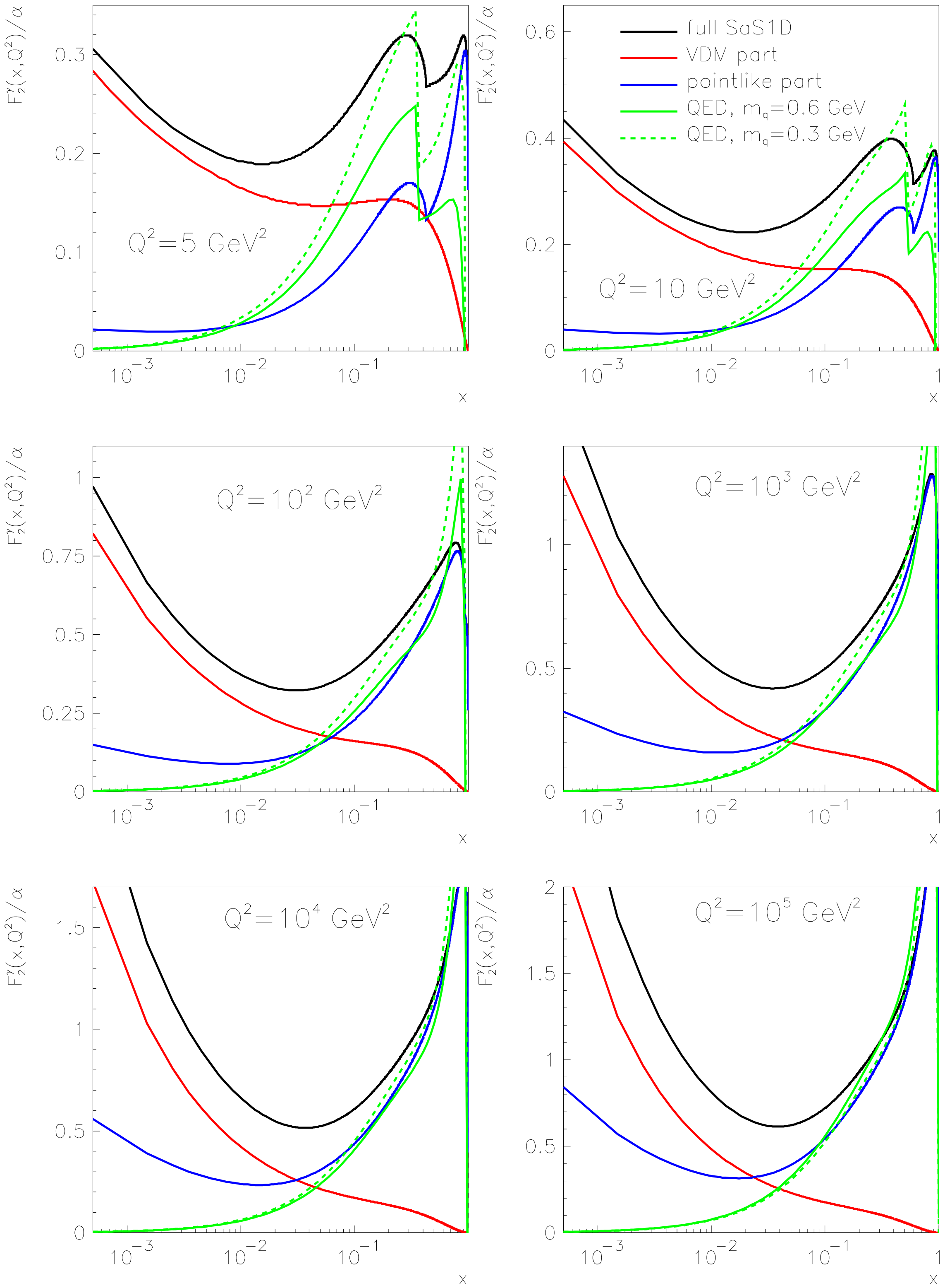




Similar plots for $F_2^\mathrm{p} \Rightarrow$ (show Figs.)


a(x)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
QED, $n_f=4$
QED, $n_f=3$
0
0.2
0.4
0.6
0.8
1
x


$a(x)-a^{QED}(x)$
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
0
0.2
0.4
0.6
0.8
1
x


full SaS1D
VDM part
pointlike part
QED, m_q=0.6 GeV
QED, m_q=0.3 GeV
Q²=5 GeV²
Q²=10 GeV²
Q²=10² GeV²
Q²=10³ GeV²
Q²=10⁴ GeV²
Q²=10⁵ GeV²
F₂^γ(x,Q²)/α
x

Alternative approach in NS channel: definition of the LO

Define first the **QED** contribution to $F_{\rm NS}^{\gamma}$

$$F_{\rm NS,QED}^{\gamma}(Q^2) = q_{\rm QED}(M) + \frac{\alpha}{2\pi} C_{x,\gamma}^{(0)}(Q/M)$$
$$q_{\rm QED}(M) \equiv \frac{\alpha}{2\pi} k_q^{(0)} \ln \frac{M^2}{M_0^2}$$

The pointlike part of quark distribution function

$$q^{\rm PL}(M) = q_{\rm QED}(M) + q_{\rm QCD}(M)$$

satisfies evolution equation with $k_q^{(0)}$, $P_{qq}^{(0)}$ **and** $k_q^{(1)}$.
Rewrite $F(Q^2)$ as the sum of its QED and QCD parts

$$F_{\rm NS}^{\gamma}(Q^2) = \underbrace{q_{\rm QED} + \frac{\alpha}{2\pi} C_{\gamma}^{(0)}}_{\rm A_0;\ pure\ QED} + \underbrace{q_{\rm QCD} + \frac{\alpha_s}{2\pi} C_q^{(1)} q_{\rm QED} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} C_{\gamma}^{(1)}}_{\equiv A_1,\ \rm starting\ as\ O(\alpha\alpha_s)} + \underbrace{\frac{\alpha_s}{2\pi} C_q^{(1)} q_{\rm QCD} + \frac{\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2 C_{\gamma}^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_q^{(2)} q_{\rm QED}}_{\equiv A_2,\ \rm starting\ as\ O(\alpha\alpha_s^2)}$$

The LO QCD correction to $F_{\rm NS}^{\gamma}(Q^2)$ is identified with

$$F_{\rm NS,LO}^{\gamma}(Q^2) = q_{\rm QCD} + \frac{\alpha_s}{2\pi} C_q^{(1)} q_{\rm QED} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} C_{\gamma}^{(1)}$$

whereas in the conventional approach

$$F_{\rm NS,LO}^{\gamma}(Q^2) = q_{\rm LO}$$

The difference concerns *semantics* (the conventional approach includes also the **QED** part) as well as *substance*. To see the latter construct the sum

$$F^{\gamma}_{\mathrm{NS,QED}} + F^{\gamma}_{\mathrm{NS,LO}} = \\ q_{\mathrm{QED}} + q_{\mathrm{QCD}} + \frac{\alpha}{2\pi} C^{(0)}_{\gamma} + \frac{\alpha_s}{2\pi} C^{(1)}_{q} q_{\mathrm{QED}} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} C^{(1)}_{\gamma}$$

which differs from that of the conventional approach

- by the **absence** of photonic c. f. $C^{(0)}_{\gamma}$ and $C^{(1)}_{\gamma}$,
- by the **absence** of the convolution $q_{\mathrm{QED}} \otimes C^{(1)}_{q}$
- by the fact $k^{(1)}_{q}$ **is included** in the evolution equation for $q(M)$.

These differences are important, but as all quantities are known, there is **no obstacle** to performing **LO QCD** analysis in the alternative approach.
On the other hand, the **NLO QCD** analysis requires so far **uncalculated** quantities.

Note: within the conventional approach $C^{(0)}_{\gamma}$ and $C^{(1)}_{q}$ enter the NLO expression

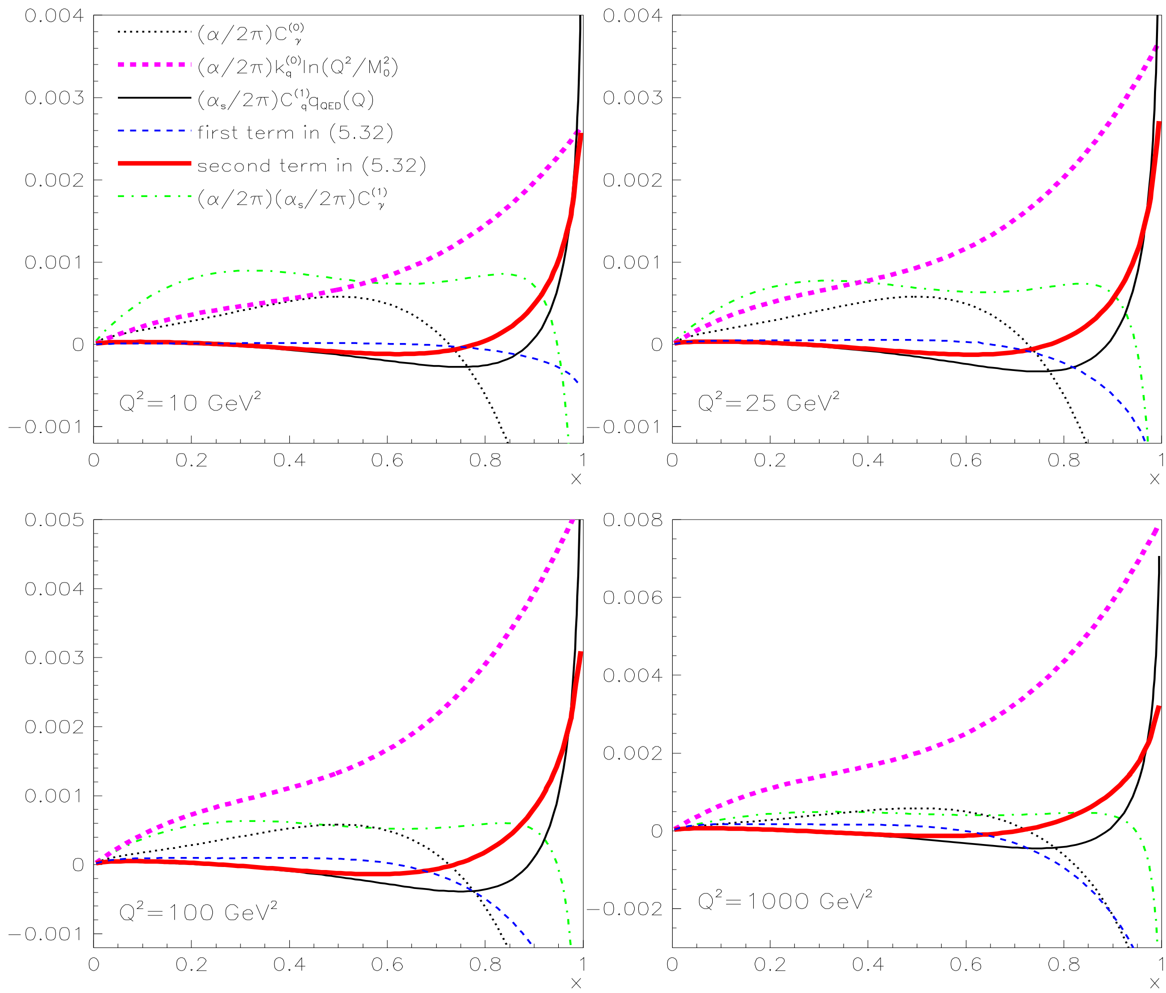
$$\frac{1}{x} F^{\gamma}_{\mathrm{NS,NLO}} = q + \frac{\alpha_s}{2\pi} C^{(1)}_{q} q + \frac{\alpha}{2\pi} C^{(0)}_{\gamma} \qquad (1)$$

but $C^{(1)}_{\gamma}$, though known, **is not used** even at NLO!

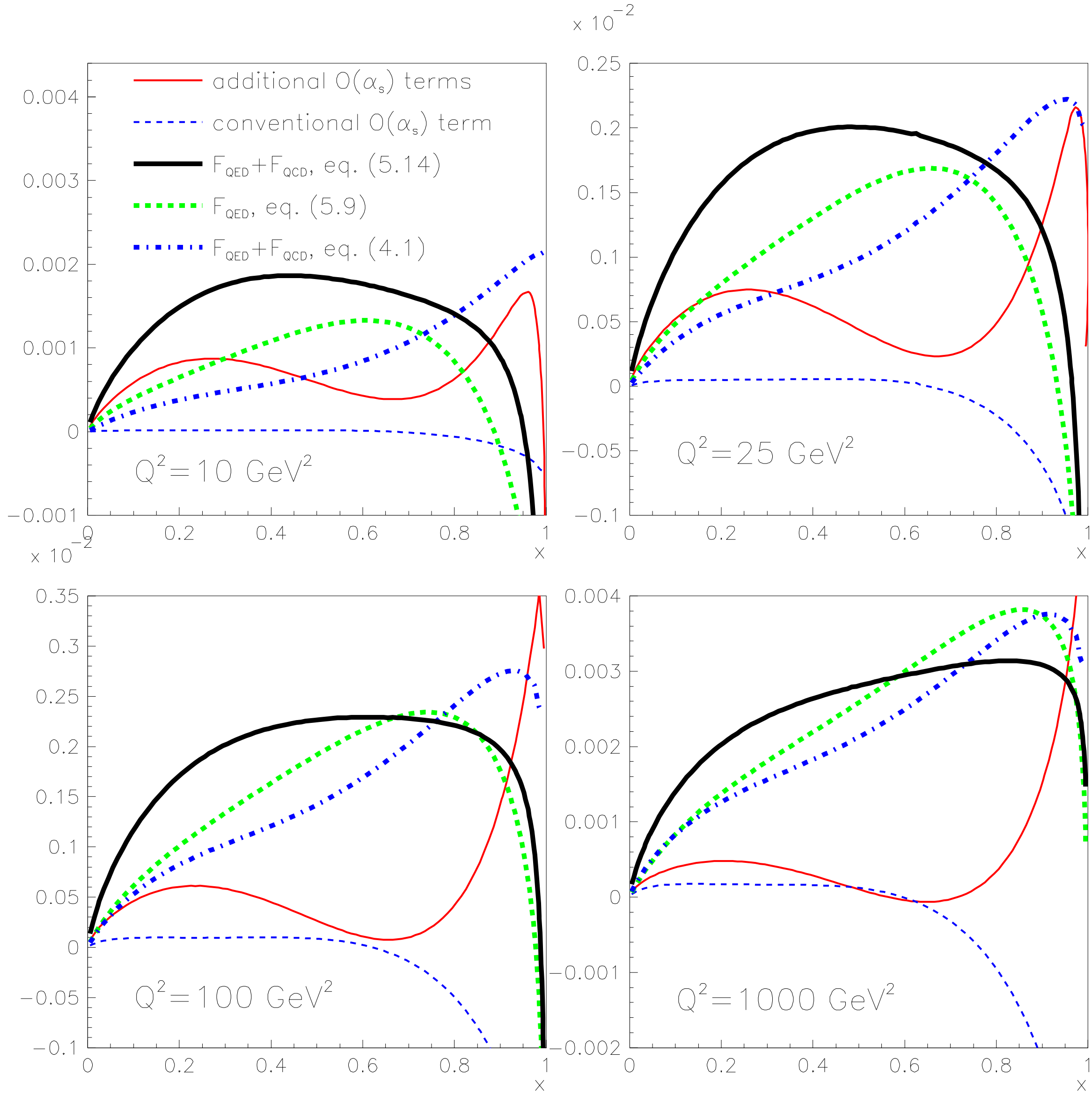
Numerical results

Calculations proceed in four stages:

- solve **analytically** evolution equations in momentum space taking into account $k_q^{(0)}$, $k_1^{(1)}$, $P_{qq}^{(0)}$
- Convert the results into the x-space using numerical **inverse Mellin** transformation
- perform in x-space the convolution $q \otimes C_1^{(1)}$
- add in x-space the contributions of $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$



Comparison of individual contributions as well as of the full expressions for $F_{\rm NS}^{\gamma}$ in the two approaches



reveals **phenomenological importance** of terms proportional to $C_{\gamma}^{(1)}$ and $k_q^{(1)}$, **both of them absent** in the conventional one.

Conclusions and outlook

1. The proposed approach to QCD analysis of F_2^γ **differs substantially** from the conventional one. It satisfies factorization scale invariance in a way that **does not rely** on physically untenable assumption $q = \mathcal{O}(\alpha/\alpha_s)$.

2. To be useful for phenomenological applications the proposed approach needs to be further elaborated by
   - extending it to the **singlet sector**
   - merging it with the **hadronic contributions**

   Work on this is in progress.

3. The **NLO QCD** analysis requires several so far unknown quantities and is thus currently **impossible to perform.** In view of the quality and number of experimental data on F_2^γ, this is at the moment no serious drawback.